Adding and Subtracting Fractions with Common Denominators

Power Up F

facts

a. **Measurement**: The ceiling is 380 cm high. Round 380 cm to the nearest hundred centimeters.

b. **Time**: One week is how many hours? (Think: $24 \times 7$.)

c. **Number Sense**: $8 \times 800$

d. **Money**: $10\cent \times 25$

e. **Fractional Parts**: $\frac{1}{2}$ of 15

f. **Percent**: 50% of 10¢

g. **Percent**: 10% of 10¢

h. **Calculation**: $6 \times 7, -2, \div 5, +1, \div 3, -3$

mental math

problem solving

Choose an appropriate problem-solving strategy to solve this problem. In 2006 postage rates for first-class mail were as follows:

| 1st ounce | 39¢ |
| 2nd ounce | 63¢ |
| 3rd ounce | 87¢ |
| 4th ounce | $1.11 |
| 5th ounce | $1.35 |
| 6th ounce | $1.59 |

Look for a pattern in the rates. How much was charged for each ounce over the 1st ounce? At this rate, what do you think the postage for an 8-ounce item would have been?
We may use fraction manipulatives to help us add and subtract fractions.

**Example 1**

**Model** Use your fraction manipulatives to illustrate this addition. Then write a number sentence for the addition.

\[
\frac{2}{4} + \frac{1}{4}
\]

Using the manipulatives, we form the fractions \(\frac{2}{4}\) and \(\frac{1}{4}\).

To add the two fractions, we combine them. We see that \(\frac{2}{4}\) plus \(\frac{1}{4}\) makes \(\frac{3}{4}\).

\[
\frac{2}{4} + \frac{1}{4} = \frac{3}{4}
\]

two fourths + one fourth = three fourths

Notice that the denominators of the fractions we added, \(\frac{2}{4}\) and \(\frac{1}{4}\), are the same. Fractions with the same denominators are said to have **common denominators**. When fractions have common denominators, we can add or subtract the fractions by simply adding or subtracting the numerators. We do not add or subtract the denominators.

\[
\frac{2}{4} + \frac{1}{4} = \frac{3}{4}
\]

Add the numerators.
Leave the denominators unchanged.

**Example 2**

**Model** Use your fraction manipulatives to illustrate this subtraction. Then write a number sentence for the subtraction.

\[
\frac{7}{10} - \frac{4}{10}
\]

We can write this expression using decimals: \(0.7 - 0.4\).
We form the fraction \( \frac{7}{10} \).

Then we remove \( \frac{4}{10} \). We see that \( \frac{3}{10} \) remains.

Start with \( \frac{7}{10} \).

Remove \( \frac{4}{10} \).

\[
\frac{7}{10} - \frac{4}{10} = \frac{3}{10}
\]

seven tenths – four tenths = three tenths

Connect Use decimal numbers to write a subtraction equation and a related addition equation for these fractions.

Example 3

Bach Yen mixed \( 1\frac{1}{4} \) cups of juice concentrate with \( 1\frac{2}{4} \) cups of water. Find the total amount of juice by adding \( 1\frac{1}{4} \) and \( 1\frac{2}{4} \).

To add mixed numbers, we add whole numbers to whole numbers and fractions to fractions. The whole numbers in this addition are 1 and 1. We add them and get 2. The fractions are \( \frac{1}{4} \) and \( \frac{2}{4} \). We add them and get \( \frac{3}{4} \).

\[
\frac{1}{4} + \frac{2}{4} = \frac{3}{4}
\]

Bach Yen made \( 2\frac{3}{4} \) cups of juice.
Example 4

There were \(2\frac{1}{2}\) chicken potpies in the refrigerator. Tupac ate \(1\frac{1}{2}\) potpies. To find the number of potpies left in the refrigerator, subtract: \(2\frac{1}{2} - 1\frac{1}{2}\).

We start with \(2\frac{1}{2}\).

We take away \(1\frac{1}{2}\). What is left is 1.

\[
2\frac{1}{2} - 1\frac{1}{2} = 1
\]

There is 1 chicken potpie left in the refrigerator.

Example 5

Federico estimates that it will take \(2\frac{1}{2}\) hours to finish reading a book and \(1\frac{1}{2}\) hours to write a book report. To find the amount of time he needs to finish the assignment, add: \(2\frac{1}{2} + 1\frac{1}{2}\).

The sum is \(3\frac{1}{2}\). The fraction \(\frac{2}{2}\) is two halves, which is one whole. So \(3\frac{1}{2}\) is \(3 + 1\), which is 4. Federico will need 4 hours to complete his assignment.

Justify Explain why the answer is reasonable.

Lesson Practice

Illustrate each addition or subtraction and write a number sentence for each problem. You may use your fraction manipulatives.

a. \(\frac{1}{10} + \frac{2}{10}\)

b. \(\frac{3}{4} - \frac{2}{4}\)

c. \(1\frac{1}{2} + 1\frac{1}{2}\)

d. \(3\frac{4}{10} - 1\frac{1}{10}\)
**1. Represent** Draw a pair of horizontal parallel line segments. Make the upper segment longer than the lower segment. Connect the ends of the segments to form a quadrilateral.

2. If a spinach pie is cut into 10 equal pieces, then what is one piece written as a fraction and as a decimal?

3. What year was two centuries after 1492?

**Formulate** For problems 4–6, write an equation and find the answer.

4. The population of Colville was 340 less than the population of Sonora. The population of Colville was 4360. What was the population of Sonora?

5. Jayne bought vegetable plants for her garden. She bought three flats of plants. There were six plants in each flat. How many plants did Jayne buy?

6. Leah is traveling 805 kilometers from Alaska to Washington state. She has already traveled 250 kilometers of that distance. How many kilometers must Leah still travel?

7. \[\frac{3}{10} - \frac{1}{2}\]

8. \[\frac{5}{10} + \frac{4}{10}\]

9. \[\frac{1}{2} - \frac{1}{2}\]

10. \[2\frac{1}{4} + 3\frac{2}{4}\]

11. **Represent** To what mixed number is the arrow pointing?

12. \[3784 + 2693 + 429 + 97 + 856 + 907\]

13. \[3106 - 528\]

14. \[$80.00 - $77.56\]
15. \(804 \times 700\)  
(29)

16. \(60 \times 43 \times 8\)  
(18, 29)

17. \(4w = 4008\)  
(26, 34)

18. \(4228 \div 7\)  
(34)

19. \(9635 \div 8\)  
(34)

20. \$7.98 \div 6\)  
(26)

21. \$10 − (\$4.56 + \$3 + \$1.29)\)  
(13, 24)

22. Round 98 to the nearest ten.  
(33)

23. **Represent** Draw an obtuse triangle.

24. One fifth of the 30 students in the class were left-handed. How many of the students were left-handed?  
(Inv. 3)

25. Xavier is learning to play the piano. Yesterday he began practice at 5:36 p.m. and finished 30 minutes later. What time did Xavier finish playing the piano?  
(28)

*26. Four friends entered a nine-mile relay race. Each person ran one fourth of the distance. How many miles did each person run?  
(40)

*27. **Represent** Draw two circles of the same size. Shade \(\frac{1}{2}\) of one circle and \(\frac{2}{3}\) of the other circle. Then compare these fractions: \(\frac{1}{2} \bigcirc \frac{2}{3}\)  
(35)

*28. **Multiple Choice** Which of these angles appears to be a 90° angle?  
(31)

29. Compare: \(\frac{1}{4}\) of 100 \bigcirc 100 ÷ 4  
(Inv. 2)

30. **Estimate** A distance of 2 feet is about the same distance as 61 centimeters. What whole number of centimeters is a reasonable estimate of the number of centimeters in 1 foot? Explain your estimate.
• Short Division
• Divisibility by 3, 6, and 9

Power Up

facts

Power Up D

count aloud

Count by 3s from 3 to 36. Count by 6s from 6 to 36.

mental math

a. **Estimation:** Round $42 to the nearest ten dollars.

b. **Measurement:** The laundry washer weighs 240 lb. The dryer weighs 160 lb. Together, how much do the washer and dryer weigh?

c. **Number Sense:** $\frac{1}{2} + \frac{1}{2}$

d. **Number Sense:** $\frac{1}{2} - \frac{1}{2}$

e. **Time:** Eight days is how many hours? (Think: $8 \times 24$.)

f. **Percent:** 50% of $100.00$

g. **Percent:** 25% of $100.00$

h. **Calculation:** $8 \times 8, -1, \div 9, +1, \div 8$

problem solving

Choose an appropriate problem-solving strategy to solve this problem. One phone number is 555-6543. How many different phone numbers in the same area code could begin with 555- and end with any arrangement of all the digits 3, 4, 5, and 6?

New Concepts

Short Division

We have learned a division algorithm in which we follow four steps: divide, multiply, subtract, and bring down. This algorithm is sometimes called “long division.” In this lesson we will practice a shortened form of this algorithm. The shortened form is sometimes called short division.
When we do short division, we follow the four steps of long division, but we do not write every number. Instead we keep track of some numbers “in our head.” We will show this by doing the same division problem both ways.

<table>
<thead>
<tr>
<th>Long Division</th>
<th>Short Division</th>
</tr>
</thead>
<tbody>
<tr>
<td>4 ( \overline{)} ) 1 4 ( \overline{) ) 4 ( \overline{)} ) 5 6 ( \overline{)} ) 1 4 ( \overline{) ) 4 5 ( \overline{)} ) 6</td>
<td></td>
</tr>
<tr>
<td>4 ( \overline{)} ) 5 6 ( \overline{)} ) 4 ( \overline{)} ) 1 6</td>
<td></td>
</tr>
</tbody>
</table>

We begin both divisions by finding \( 4 \overline{)} 5 \). We write “1” above the 5 and then we multiply. In short division we keep the multiplication answer in our head. Then we subtract. **In short division we write the subtraction answer in front of the next digit.** Here we write a small “1” in front of the 6 to make 16. In short division we do not bring down this digit; instead we “bring up” the subtraction answer. Now we find \( 4 \overline{)} 1 6 \) and write “4” above the 6. We multiply and subtract in our head and find that there is no remainder.

**Example 1**

A total of 840 people attended the 5 performances of the school play. Find the average number of people who attended each performance by dividing 840 by 5.

We will use short division to find the answer. We write the numbers in a division box and then we divide and write “1” above the 8. Then we multiply and subtract in our head to get 3. We bring up the 3 and write it in front of the next digit. Next we find \( 5 \overline{)} 3 4 \). We continue to divide, multiply, subtract, and bring up. We bring up the last subtraction answer as the remainder. The answer is 168.

\[
\begin{array}{c}
1 \\
6 \\
8 \\
4 \\
0
\end{array}
\]

An average of **168 people** attended each performance of the school play.

**Verify** How can we check the answer?

**Divisibility by 3, 6, and 9**

**Divisibility** is the “ability” of a number to be divided by another number without a remainder. In Lesson 22 we found that whole numbers ending in an even digit are divisible by 2. Whole numbers ending in zero are divisible by 2, 5, and 10, and whole numbers ending in 5 are divisible by 5.
In this lesson we will learn to identify whole numbers divisible by 3, 6, and 9. We need to look at all of the digits of a whole number to decide whether the number is divisible by 3, 6, or 9. In fact, we add the digits of the number. If the sum of the digits is divisible by 3, then the number is divisible by 3. If the sum of the digits is divisible by 9, the number is divisible by 9. Let’s consider the number 438.

\[ 438 \rightarrow 4 + 3 + 8 = 15 \]

The sum of the digits is 15. Fifteen can be divided by 3 without a remainder, so 438 is divisible by 3. However, 15 cannot be divided by 9 without a remainder, so 438 is not divisible by 9.

A number is divisible by 6 if the number is even (divisible by 2) and divisible by 3. Since 438 is even and divisible by 3, it is also divisible by 6. Below we divide 438 by 3, 6, and 9 to show that 438 is divisible by 3 and 6 but not by 9.

<table>
<thead>
<tr>
<th>146</th>
<th>73</th>
<th>48 R 6</th>
</tr>
</thead>
<tbody>
<tr>
<td>3 (\overline{438})</td>
<td>6 (\overline{438})</td>
<td>9 (\overline{438})</td>
</tr>
<tr>
<td>3</td>
<td>42</td>
<td>36</td>
</tr>
<tr>
<td>(\bar{13})</td>
<td>18</td>
<td>(\bar{78})</td>
</tr>
<tr>
<td>(\bar{12})</td>
<td>18</td>
<td>72</td>
</tr>
<tr>
<td>(\bar{18})</td>
<td>0</td>
<td>(\bar{6})</td>
</tr>
<tr>
<td>(\bar{18})</td>
<td>0</td>
<td>(\bar{6})</td>
</tr>
</tbody>
</table>

The following table summarizes the divisibility rules for 3, 6, and 9:

**Divisibility Tests for 3, 6, and 9**

A number is divisible by …

3 if the sum of its digits is divisible by 3.

6 if the number is divisible by 2 and 3.

9 if the sum of its digits is divisible by 9.

---

**Math Language**

When a number is divisible by 2, it means that 2 is a factor of that number. When a number is divisible by 3, it means that 3 is a factor of that number, and so on.

The numbers 2, 3, and 6 are factors of 438 because 438 is divisible by 2 and by 3.
Which of these numbers is divisible by 3, by 6, and by 9?

A 456  B 567  C 576  D 645

We add the digits of each number and find whether the sums are divisible by 3 and by 9. We note that 456 and 576 are even.

\[
\begin{align*}
456 &\rightarrow 4 + 5 + 6 = 15 \text{ divisible by 3 and 6} \\
567 &\rightarrow 5 + 6 + 7 = 18 \text{ divisible by 3 and 9} \\
576 &\rightarrow 5 + 7 + 6 = 18 \text{ divisible by 3, 6, and 9} \\
645 &\rightarrow 6 + 4 + 5 = 15 \text{ divisible by 3}
\end{align*}
\]

Only 576 is divisible by 3, by 6, and by 9.

Lesson Practice

Divide using short division:

a. \(3\overline{)435}\)  b. \(6\overline{)534}\)  c. \(9\overline{)567}\)

d. \(4\overline{)500}\)  e. \(7\overline{)800}\)  f. \(10\overline{)836}\)

g. \(5\overline{)600}\)  h. \(3\overline{)616}\)  i. \(6\overline{)858}\)

For problems j–o, decide whether the number is divisible by 3, by 6, by 9, or by none of these numbers.

j. 1350  k. 4371  l. 1374

m. 436  n. 468  o. 765

Written Practice

*1. Represent  
Draw a pentagon.

Formulate  
For problems 2–4, write an equation and find the answer.

*2. A rattlesnake’s rattle shakes about 50 times each second. At that rate, how many times would it shake in 1 minute?

\[60 \times 50 = t; \text{3000 times}\]

3. One afternoon in Tucson, Arizona, the temperature was 98°F. One hour later, the temperature was 101°F. By what number of degrees did the temperature change during that hour?

\[101 - 98 = d; \text{the temperature increased 3°F.}\]

4. During one week, 124 books were checked out of the school library. That same week, 77 books were returned. Did the library have more books or fewer books at the end of the week than at the beginning of the week? How many more or fewer?
5. Sh'Tania cut a 21-foot long ribbon into four equal lengths. How many feet long was each length of ribbon?

*6. Represent (39) Draw two rectangles of the same size. Shade \( \frac{3}{4} \) of one rectangle and \( \frac{3}{5} \) of the other rectangle. Then compare these fractions:

\[
\frac{3}{4} \, \bigcirc \, \frac{3}{5}
\]

7. \( \frac{3}{10} + \frac{4}{10} \)  
8. \( 1\frac{1}{3} + 2\frac{1}{3} \)

9. \( \frac{7}{10} - \frac{4}{10} \)  
10. \( 5\frac{1}{4} - 2\frac{1}{4} \)

*11. Connect (40) Use a mixed number to name the number of shaded circles shown below.

12. Round 151 to the nearest hundred.

13. Connect (38) To what fraction is the arrow pointing?

14. Compare: \( \frac{1}{3} \) of 30 \( \bigcirc \) \( \frac{1}{5} \) of 50

15. Two fourths of a circle is what decimal part of a circle?

16. \( $18.73 \quad $34.26 \quad $79.33 \)
17. \( 6010 \quad \frac{r}{543} \quad 47 \quad 493 \quad 71 \quad 82 \)
18. \( 936 \quad 18 \quad 47 \quad 493 \quad 71 \quad 82 \)
19. \( 346 \quad \times \quad 80 \)
20. \( $7.25 \quad \times \quad 90 \)
21. \( 670 \quad \times \quad 700 \)
Divide using short division:

22. \( 4 \overline{) 1736} \)

23. \( 8 \overline{) $17.60} \)

24. \( 3 \overline{) 100} \)

25. At 7:00 a.m. the temperature was 16°C. By 3 p.m. the temperature had increased 5°C. By 11:00 p.m. the temperature had decreased 2°C. What was the 11:00 p.m. temperature?

*26. Multiple Choice* Which of these angles appears to be an acute angle?

A \[
\hspace{1cm}
\]

B \[
\hspace{1cm}
\]

C \[
\hspace{1cm}
\]

D \[
\hspace{1cm}
\]

27. **Multiple Choice** Which of these numbers is divisible by 3, by 6, and by 9?

A 369  
B 246  
C 468  
D 429

28. \( 3 + (4 + 5) = (3 + 4) + m \)

**29. Analyze** Two twelfths of the names of the months begin with the letter A, and three twelfths begin with the letter J. What fraction of the names of the months begin with either A or J?

30. **Justify** Renaldo estimated the quotient of 431 \( \div 7 \) to be about 60. Did Renaldo make a reasonable estimate? Explain why or why not.

**Real-World Connection** It is Regina’s second day at a new school. She doesn’t remember her locker number. She knows it is either number 468, 465, or 456. If Regina’s locker number is divisible by 3 and by 6 but not by 9, which locker is hers?
• More Arithmetic with Mixed Numbers

Power Up

facts  
Power Up E

count aloud

Count by 7s from 7 to 84.

mental math

a. **Estimation**: The dining room table is 162 cm long. Round the length of the table to the nearest ten centimeters.
b. **Money**: Which coin has a value of 10% of a dollar?
c. **Money**: Which coin has a value of 25% of a dollar?
d. **Number Sense**: \( \frac{2}{3} + \frac{1}{3} \)
e. **Number Sense**: \( \frac{2}{3} - \frac{1}{3} \)
f. **Fractional Parts**: \( \frac{1}{2} \) of 9
g. **Percent**: 25% of the 16 students had blue eyes. How many students had blue eyes?
h. **Percent**: 50% of 50

problem solving

Choose an appropriate problem-solving strategy to solve this problem. In the game tic-tac-toe, the goal is to get three Xs or three Os in a row. How many ways can three Os be placed in tic-tac-toe? Draw a diagram to find all the ways to get three Os in a row. Shidoshi says that there are 9 ways to do this. Is he correct?

New Concept

In Lesson 40 we studied some word problems that had mixed-number answers. We found that the remainder in a division problem can be divided to result in a fraction. In this lesson we will continue our study.
Example 1

The local pizzeria will donate 14 pizzas to the sixth-grade picnic. How many pizzas will there be for each of the three classes of sixth graders?

Fourteen pizzas will need to be divided into three equal groups, so we divide 14 by 3.

\[
\begin{array}{c|c}
3 & 14 \\
3 & \underline{12} \\
\hline
2 & \underline{2} \\
\end{array}
\]

There are four whole pizzas for each class.

Two pizzas remain to be divided.

Four pizzas for each class is 12 pizzas. The remainder shows us there are still 2 pizzas to divide among the three classes. If each of the remaining pizzas is divided into 3 parts, there will be 6 parts to share. Each class can have 2 of the parts.

There will be \(4\frac{2}{3}\) pizzas for each class.

Notice that the 2 in \(\frac{2}{3}\) is the remainder and the 3 in \(\frac{2}{3}\) is the divisor.

Example 2

At home, Mario uses a bread machine to make bread. To make 3 loaves of bread, Mario uses 7 cups of whole wheat flour. Which quotient represents the number of cups of flour Mario uses to make one loaf of bread?

\[
\begin{array}{c|c}
3 & 7 \\
3 & \underline{6} \\
\hline
1 & \underline{1} \\
\end{array}
\]

\[
\begin{array}{c|c}
3 & \frac{1}{3} \\
3 & \underline{6} \\
\hline
1 & \underline{1} \\
\end{array}
\]

Since the 7 cups of flour for 3 loaves of bread is divided into 3 equal parts, there are \(2\frac{1}{3}\) cups of whole wheat flour in each loaf.
Example 3

A whole circle is 100% of the circle. If a circle is divided into thirds, then each third is what percent of the whole circle?

We divide 100% by 3 to find the percent for each third.

\[
\begin{array}{c}
33 \\
3) 100 \\
\underline{9} \\
10 \\
\underline{9} \\
1 \\
\end{array}
\]

1% remains to be divided.

If each third of the circle were 33%, then the total would be 99%. However, the total needs to be 100%, so we need to divide the remaining 1% by 3. One divided by three is \(\frac{1}{3}\). We write “\(\frac{1}{3}\)” after the 33. Each third of the circle is \(33\frac{1}{3}\)% of the whole circle.

\[
\begin{array}{c}
33\frac{1}{3} \\
3) 100 \\
\underline{9} \\
10 \\
\underline{9} \\
1 \\
\end{array}
\]

We have studied whole numbers, fractions, and mixed numbers. When adding these numbers, we add whole numbers to whole numbers and fractions to fractions. When subtracting, we subtract whole numbers from whole numbers and fractions from fractions. Remember that order matters in subtraction.

Example 4

A recipe calls for adding \(1\frac{1}{2}\) cups of milk to 5 cups of whole wheat flour. To find the sum of these ingredients, add 5 and \(1\frac{1}{2}\).

We add whole numbers to whole numbers and fractions to fractions. The sum of the whole numbers 5 and 1 is 6. There is no fraction to add to \(\frac{1}{2}\). So 5 cups plus \(1\frac{1}{2}\) cups is \(6\frac{1}{2}\) cups.
Example 5

For Tuesday night's homework, Ali spent 1 hour reading and \( \frac{1}{2} \) hour finishing a math assignment. To find the total time Ali spent on his homework, add: \( 1 + \frac{1}{2} \).

We add whole numbers to whole numbers and fractions to fractions. There is no fraction to add to \( \frac{1}{2} \) and no whole number to add to 1. We write the whole number and fraction together to make the mixed number \( 1 \frac{1}{2} \). We find that Ali spent \( 1 \frac{1}{2} \) hours on his homework.

Example 6

Graciela bought \( 3 \frac{1}{2} \) pounds of grapes at the store and used \( \frac{1}{2} \) pound of grapes in a fruit salad. To find the weight of the grapes she has left, perform this subtraction: \( 3 \frac{1}{2} - \frac{1}{2} \).

When we subtract the fractions, we find the answer is \( 3 \), which is zero. So subtracting \( \frac{1}{2} \) pound from \( 3 \frac{1}{2} \) pounds leaves \( 3 \) pounds of grapes.

Lesson Practice

a. **Represent**  Draw a diagram to illustrate this story. Then find the answer using pencil and paper. Follow Example 1 in the lesson.

Anil’s Pizza Shop will donate 15 pizzas to the fifth grade picnic. How many pizzas will there be for each of the four classes of fifth graders?

b. A whole circle is divided into sevenths. Each seventh is what percent of the whole circle?

Find each sum or difference:

c. three and two fourths plus one fourth

\[
\begin{array}{c}
\text{\begin{figure}[h!]
\centering
\includegraphics[width=0.3\textwidth]{image1.png}
\end{figure}}
\end{array}
\]

d. three and two fourths minus one fourth

\[
\begin{array}{c}
\text{\begin{figure}[h!]
\centering
\includegraphics[width=0.3\textwidth]{image2.png}
\end{figure}}
\end{array}
\]

e. \( 6 \frac{2}{3} - 3 \)

f. \( 3 \frac{1}{2} + 2 \)

g. \( 2 \frac{1}{2} - \frac{1}{2} \)

h. \( \frac{3}{4} + 2 \)

i. **Formulate**  Write a word problem for this subtraction: \( 2 \frac{1}{2} - 1 \frac{1}{2} \)
1. **Represent** Draw a pair of vertical parallel line segments of the same length. Connect the ends of the segments to make a quadrilateral.

2. **Formulate** For problems 2–4, write an equation and find the answer.

2. Angela poured 32 ounces of juice equally into 4 cups. How many ounces of juice were in each cup?

3. A stick 100 centimeters long broke into two pieces. One of the pieces was 48 centimeters long. How long was the other piece?

4. Joshua has $28.75. How much more money does he need to buy a skateboard that costs $34.18?

5. **Represent** Draw a square. Shade all but one fourth of it. What percent of the square is not shaded?

6. Round 158 to the nearest ten.

7. \[5 + 2\frac{1}{2}\]

8. \[12\frac{1}{2} + 12\frac{1}{2}\]

9. \[1 + \frac{1}{3}\]

10. \[3\frac{1}{2} - \frac{1}{2}\]

11. \[4\frac{3}{5} - 3\frac{1}{5}\]

12. \[5\frac{3}{8} - 1\]

13. A whole circle is divided into eighths. Each eighth is what percent of the whole circle?

14. How many eighths are equal to one half?

15. \[408 \times 70\]

16. \[$9.67 \times 60\]

17. \[970 \times 900\]

18. \[$3.47 + $5.23 + $7.68 + $2.42\]

19. \[r - 3977 = 309\]

20. \[9013 - w = 3608\]
21. \(7)\overline{890}\) 
22. \(6)\overline{100}\) 
23. \(4)\overline{8035}\)

24. How many minutes is one tenth of an hour?

25. The highest temperature ever recorded in Anchorage, Alaska, was 85°F. In Los Angeles, California, the highest temperature ever recorded was 110°F. How many degrees cooler is a temperature of 85°F than a temperature of 110°F?

26. (38) Connect What mixed number names point \(Y\) on this number line?

27. a. A right angle measures how many degrees?
   b. Half of a right angle is how many degrees?
   c. Use the numbers in the answers to parts a and b to write a fraction equal to \(\frac{1}{2}\).

28. Multiple Choice Which of these numbers is divisible by 6 and by 10?
   A 610  B 510  C 410  D 312

29. What number is \(\frac{1}{8}\) of 1000?

30. This table shows the average depth of three different bodies of water:

<table>
<thead>
<tr>
<th>Body of Water</th>
<th>Average Depth (in feet)</th>
</tr>
</thead>
<tbody>
<tr>
<td>North Sea</td>
<td>308</td>
</tr>
<tr>
<td>Persian Gulf</td>
<td>328</td>
</tr>
<tr>
<td>Hudson Bay</td>
<td>305</td>
</tr>
</tbody>
</table>

   a. Arrange the names of the bodies of water in order from shallowest average depth to deepest.

   b. Explain What operation would you choose to find how much deeper the average depth of the Persian Gulf is than the average depth of the North Sea? Explain why you chose that operation.

   c. Explain Which body or bodies of water have an average depth of about 300 feet? Explain your answer.
• Measuring Lengths with a Ruler

Power Up

facts

mental math

a. **Time**: How many minutes is 5 hours?
b. **Time**: What time is 33 minutes after 6:27 a.m.?
c. **Money**: Which bill has a value of 25% of a $20 bill?
d. **Number Sense**: $\frac{3}{4} + \frac{1}{4}$
e. **Number Sense**: $\frac{3}{4} - \frac{1}{4}$
f. **Geometry**: The sides of a triangle are 3 cm, 4 cm, and 5 cm long. What is the distance around the triangle?
g. **Measurement**: 17 yards is how many feet? *(Think: $17 \times 3$.*
h. **Calculation**: $2 \times 25, -1, \div 7, +1, \div 2, \times 5, +1, \div 3$

problem solving

Choose an appropriate problem-solving strategy to solve this problem. Ms. Gordy wants to arrange the 24 classroom desks in equal-length rows. Each row must contain at least 4 desks but not more than 8 desks. How many different arrangements of desks can she make? Describe the possible arrangements.

New Concept

Reading Math

A centimeter is about the width of the fingernail on your index finger.

You might have a ruler at your desk that has both a centimeter scale and an inch scale. We will practice using both scales in this lesson to measure length. **Length** is the measure of the distance between two points. On the next page we show a centimeter scale and a millimeter scale. The words **centimeter** and **millimeter** are abbreviated “cm” and “mm,” respectively.
The centimeter scale is divided into segments 1 centimeter long and may be further divided into millimeters (mm). Notice that **10 millimeters equals 1 centimeter**. The arrow is 4 centimeters long. It is also 40 millimeters long.

**Example 1**

The distance across a nickel is about 2 centimeters. Two centimeters is how many millimeters?

We remember that 1 centimeter equals 10 millimeters, so 2 centimeters equals **20 millimeters**.

**Analyze** What is the length, in millimeters, of ten nickels placed end to end?

**Example 2**

What is the length of the rectangle below?

There are tick marks on the scale to mark each millimeter. Notice that the tick marks for every fifth and tenth millimeter are lengthened to make the scale easier to read. We see that the rectangle’s length is 20 millimeters plus 5 more millimeters, which is **25 mm**.

Centimeters and millimeters are units of length in the **International System of Units**, which is sometimes called the **metric system**. The basic unit of length in the metric system is a meter. You might have a meterstick in your classroom. When you take a big step, you move about one meter. A centimeter is \( \frac{1}{100} \) of a meter, and a millimeter is \( \frac{1}{1000} \) of a meter. Units in the metric system are related by the number 10.
Inches, feet, yards, and miles are not units in the metric system. These units of length are part of the **U.S. Customary System**. Units in the U.S. Customary System are not related by the number 10. Instead, 12 inches equals a foot, 3 feet equals a yard, and 5280 feet (or 1760 yards) equals a mile. Inches usually are not divided into tenths; rather, they are divided into halves, fourths, eighths, sixteenths, and so on. When we abbreviate the word *inch* or *inches*, we make sure to include a period in our abbreviation. We abbreviate *inch* or *inches* like this: *in*. Here we show an inch scale divided into eighths:

![Inch Scale](image)

Below we show a magnified portion of an inch ruler. We see that \(\frac{2}{8}\) of an inch equals \(\frac{1}{4}\) of an inch, \(\frac{4}{8}\) of an inch equals \(\frac{1}{2}\) of an inch, and \(\frac{6}{8}\) of an inch equals \(\frac{3}{4}\) of an inch.

**Math Language**

A \(\frac{1}{2}\)-inch segment is also called a *quarter inch*. How many quarter inches are equal to 1 inch?

**Example 3**

How many inches long is this arrow?

![Arrow](image)

The marks on the ruler divide each inch into eight smaller segments. Each small segment is one eighth of an inch long. Measuring the arrow, we see that its length is 2 full inches plus 4 small segments, or \(2\frac{4}{8}\) inches. However, there are other ways to name the fraction \(\frac{4}{8}\). We see that the mark at the end of the arrow is exactly halfway between 2 and 3. That mark is the two-and-one-half-inch mark, so the length of the arrow is \(2\frac{1}{2}\) inches, or \(2\frac{1}{2}\) in. Notice on the ruler that the half-inch marks are slightly longer than the quarter-inch marks and the eighth-inch marks.

**Analyze** How many \(2\frac{1}{2}\)-inch segments are equal to 10 inches? Explain how you know.
Example 4

A Mangrove Skipper butterfly can have a wingspan of \(2\frac{3}{4}\) inches. Is \(2\frac{3}{4}\) inches closer to 2 inches or to 3 inches?

Since \(\frac{3}{4}\) is more than \(\frac{1}{2}\), we know that \(2\frac{3}{4}\) inches is closer to 3 inches.

**Conclude** What types of items would you measure using:

a. inches?  
   b. centimeters?  
   c. millimeters?

Lesson Practice

Use a centimeter ruler to measure each segment in centimeters and in millimeters:

a. ____________  
b. ____________________  
c. ____________________

d. Estimate Measure the length of your math book to the nearest centimeter.

e. One centimeter is how many millimeters?

f. How many millimeters is 5 centimeters?

g. Write the abbreviations for “centimeter” and “millimeter.”

h. How many millimeters long is the nail?

i. How many centimeters long is the arrow?

Connect For problems j–o, name the mark on the ruler to which each arrow is pointing.

j. k. l. m. n. o.

Use an inch ruler to measure each segment to the nearest eighth of an inch:

p. ____________________  
q. ____________________  
r. ____________________
s. **Estimate** Is $6\frac{1}{8}$ inches closer to 6 inches or 7 inches?

t. Round $5\frac{7}{8}$ inches to the nearest inch.

---

**Written Practice**

Distributed and Integrated

**1. Represent** Draw a quadrilateral that has four right angles.

2. In her pocket Sh’Marah has 3 pennies, 2 nickels, a dime, 3 quarters, and a half dollar. How much money is in Sh’Marah’s pocket?

For problems 3–5, write an equation and find the answer.

3. One hundred thirty-eight kindergartners climbed on three buses to go to the zoo. If the same number of children were on each bus, how many children were on each bus?

4. Blanca is in third grade. The index finger of Blanca’s left hand is 6 centimeters long. What is that length in millimeters?

5. South Dakota became our 40th state in 1889. Vermont became our 14th state in 1791. How many years before South Dakota became a state did Vermont become a state?

6. Three friends want to share five oranges equally. How many oranges should each friend receive?

**7. Connect** What mixed number names point Z on this number line?

---

8. \[ \frac{33}{4} - \frac{12}{4} \]

9. \[ \frac{41}{2} - \frac{1}{2} \]

10. \[ 5\frac{1}{4} - 4 \]

11. \[ 33\frac{1}{3} + 33\frac{1}{3} \]

12. \[ 5\frac{1}{3} + 3 \]

13. \[ 8\frac{3}{8} + \frac{4}{8} \]
14. What is the length of this rectangle?

15. \(352 + 4287 + 593 + 7684 + 9856\)

16. \(3627 - 429\)

17. \(9104 - (2000 - 66)\)

18. \(491 \times 700\)

19. \(60 \times 8 \times 37\)

20. \(5n = 3175\)

21. \(2964 \div 10\)

*22. Represent Draw a circle. Shade all but one third of it. What percent of the circle is shaded?

*23. Multiple Choice When we count by tens, we find that the number 256 is closest to which of the following numbers?

A 240  B 250  C 260  D 300

24. The principal of an elementary school has scheduled a fire drill for 30 minutes after the time shown on the clock. What time is the fire drill scheduled for?

25. List Write the common factors of 50 and 100.

26. One foot is 12 inches. One fourth of a foot is how many inches?

27. Estimate A shopper purchased two items at a hardware store. The cost of each item was $3.49. Explain how to estimate the cost in dollars of the purchase.

*28. Multiple Choice Which of these numbers is divisible by 3 and by 5?

A 305  B 315  C 325  D 355
29. The recipe called for $\frac{1}{3}$ cup of vegetable oil for one batch of corn bread. How much vegetable oil is needed to bake two batches of corn bread?

30. The thermometer shows the low temperature for a winter day in International Falls, Minnesota. The high temperature that day was $13^\circ$ warmer. What was the high temperature that day?

---

**Real-World Connection**

Two pieces of wood are 108 cm long when placed end to end. One piece is 28 cm longer than the other. How many millimeters long is each piece of wood?
• Classifying Quadrilaterals

Power Up

facts

Count by 25s from 25 to 300.

a. **Money:** How many cents is 3 quarters?

b. **Money:** How many cents is 6 quarters?

c. **Estimation:** Round 278 to the nearest ten. Then add 25. What is the number?

d. **Measurement:** Darrius ran 875 meters and then walked 125 meters. How many total meters did he run and walk?

Power Up D

- **Money:** 10 × 15¢
- **Percent:** 50% of 10
- **Percent:** 10% of 50
- **Calculation:** 6 × 4, ÷ 3, + 2, ÷ 5, × 7, + 1, ÷ 3

mental math

problem solving

Choose an appropriate problem-solving strategy to solve this problem. Each package of balloons contains exactly 10 red balloons, 8 blue balloons, and 6 yellow balloons. For party decorations, J’Narra purchased two packages of balloons. If J’Narra wants to use equal numbers of each color, what is the greatest number of balloons she can use at the party? How many balloons will be unused?

New Concept

Recall from Lesson 32 that a quadrilateral is a polygon with four sides. Although all quadrilaterals have four sides, quadrilaterals have many different shapes. Here is an assortment of quadrilaterals:
We can classify (sort) quadrilaterals into different types, such as squares, rectangles, parallelograms, and trapezoids. In this lesson we will learn ways to sort quadrilaterals, and we will practice drawing named quadrilaterals.

One way quadrilaterals are sorted is by parallel sides. Recall that parallel segments run in the same direction and remain the same distance apart. Here we show three pairs of parallel segments:

```
\ /
\ /
_______
```

If we put the left-hand pair of segments with the center pair, we get this quadrilateral:

```
    /
   /|
  /  |
```

If we put the right-hand pair with the center pair, we get this quadrilateral:

```
  
```

Both of these quadrilaterals have two pairs of parallel sides. Below we show some more quadrilaterals with two pairs of parallel sides. Use a finger or the eraser of your pencil to trace the pairs of parallel sides on each quadrilateral. Notice that the two segments that form a parallel pair are the same length. These quadrilaterals are called parallelograms.

**Parallelograms**

Parallelograms are quadrilaterals with two pairs of parallel sides and are one classification of quadrilaterals.
**Trapezoids** are another type of quadrilateral. Trapezoids have only one pair of parallel sides. (The other pair of sides are not parallel.) Here are some examples of trapezoids. First use your finger to trace the parallel sides, and then trace the sides that are not parallel. Notice that the parallel segments in each figure are not the same length.

Some quadrilaterals have no parallel sides. In the United States we call these shapes **trapeziums**.

**Example 1**

**Draw an example of a parallelogram, a trapezoid, and a trapezium.**

To draw a parallelogram, we may begin by drawing two parallel segments of the same length.

```
   ...
   ...
```

Then we draw two more segments between the endpoints. We check these two segments to be sure they are parallel.

**Parallelogram**

```
   ...
```

This parallelogram happens to look like a rectangle. As we will see in a moment, rectangles are a special type of parallelogram.

To draw a trapezoid, we can begin by drawing two parallel segments of different lengths.

```
   ...
   ...
```

Then we draw two more segments between the endpoints.

**Trapezoid**
To draw a trapezium, we may begin by drawing two segments that are not parallel and do not intersect.

Then we draw two segments between the endpoints. We check that these two segments are not parallel.

There are different categories of parallelograms, trapezoids, and trapeziums. In this lesson we will look at three types of parallelograms: rectangles, rhombuses, and squares.

A parallelogram with four congruent angles is a rectangle. Each angle of a rectangle is a right angle. Each pair of adjacent sides is perpendicular.
A parallelogram with four congruent sides (four sides of equal length) is a **rhombus**. One common rhombus shape is the “diamond.” A rhombus is an equilateral quadrilateral (just like a triangle with all sides of equal length is an equilateral triangle).

![Rhombus Diagram]

Notice that a **square** is both a rectangle and a rhombus. A square has four right angles and four congruent sides.

**Analyze** How many pairs of perpendicular sides does a square have?

---

**Example 2**

This figure is an example of a square:

![Square Diagram]

**Which of the following could not be a square?**

- A parallelogram
- B rhombus
- C rectangle
- D trapezoid

A square is a quadrilateral with parallel sides of equal length that intersect at right angles. A parallelogram, a rhombus, and a rectangle could all be a square. A **trapezoid** could never be a square, so the answer is D.

---

**Lesson Practice**

**Classify** The words **parallelogram, trapezoid, trapezium, rectangle, rhombus,** and **square** were used in this lesson to describe quadrilaterals. Use every word that applies to describe each quadrilateral below.

- a. ![Parallelogram]
- b. ![Trapezoid]
- c. ![Rhombus]
- d. ![Trapezium]
- e. ![Rectangle]
- f. ![Square]

**g. Explain** Describe the difference between a parallelogram and a trapezoid.

**h. Represent** Draw a rhombus that does not have right angles.
*1. **Represent**  
(45) Draw a rectangle with all sides the same length.

**Formulate**  
For problems 2–4, write an equation and find the answer.

2. **Justify**  
(16) Olivia paid $10 and received $2.47 in change. How much money did she spend? Explain why your answer is reasonable.

3. Each of the fifty states has two U.S. senators. Altogether, how many U.S. senators are there?

4. A large community theater performed a musical. The theater was filled all 4 nights. If 2500 people attended in all, then how many people attended each night?

5. There were 8 gallons of juice for the third grade picnic. How many gallons of juice were there for each of the 3 third grade classes?

6. Ten millimeters equals how many centimeters?

7. How many inches long is this arrow?

![Arrow Image]

8. \(3 \frac{1}{3} + 1 \frac{1}{3}\)

9. \(4 \frac{1}{4} + 2\)

10. \(3 + \frac{3}{4}\)

11. \(5 \frac{3}{8} - 2\)

12. \(6 \frac{3}{4} - 1 \frac{2}{4}\)

13. \(5 \frac{1}{2} - 1 \frac{1}{2}\)

14. \(\$87.93 + \$35.16 + \$42.97 + \$68.74\)

15. \(\$50.26 - \$13.87\)

16. \(6109 - a = 4937\)

17. \(9314 \times 70\)

18. \(\$2.34 \times 600\)

19. \(4287 \times 5\)
20. \( \frac{9636}{9} \)

21. \( 8m = $34.16 \)

**22. [Represent]** Draw a rectangle and shade \( \frac{3}{5} \) of it. What percent of the rectangle is shaded?

23. Round 256 to the nearest ten.

**24. [Multiple Choice]** Which of these triangles appears to be a right triangle?

A [Diagram]

B [Diagram]

C [Diagram]

D [Diagram]

25. [Connect] To what number is the arrow pointing?

26. Use a ruler to measure the length of this segment in inches:

27. Show how to check this division answer. Is the answer correct?

28. [Represent] Draw two circles. Shade one and three fourths of them.

29. The segment from point A to point B is \( 1\frac{2}{3} \) inches long. The segment from point B to point C is \( 1\frac{3}{5} \) inches long. How long is the segment from point A to point C?

30. Use this table to answer parts a and b:

<table>
<thead>
<tr>
<th>Number of Small Apples</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Calories</td>
<td>65</td>
<td>130</td>
<td>195</td>
<td>260</td>
</tr>
</tbody>
</table>

a. [Generalize] Write a rule that describes how to find the number of calories for any number of small apples.

b. [Predict] A bag of 10 small apples contains what number of calories?
• Word Problems About a Fraction of a Group

Power Up

facts

Power Up F

mental math

a. Money: What coin has a value of 50% of 50¢?
b. Number Sense: \( \frac{5}{10} + \frac{2}{10} \)
c. Number Sense: \( \frac{5}{10} - \frac{2}{10} \)
d. Number Sense: An ant has 6 legs. Altogether, how many legs do 82 ants have? (Think: \( 6 \times 82 \).)
e. Money: One bag of apple chips costs 75¢. Ten bags of apple chips cost how much?
f. Fractional Parts: \( \frac{1}{2} \) of 51
g. Percent: What is 25% of 24 hours?
h. Calculation: \( 10 \times 10, \div 2, -1, \div 7, -1, \div 3, -2 \)

problem solving

Choose an appropriate problem-solving strategy to solve this problem. Renee counted boats on the lake. One half of the boats were sailboats. Four of the boats were rowboats. The remaining boats were motorboats. If there were 12 boats altogether, how many boats were motorboats?

New Concept

One type of “equal groups” problem is “fraction-of-a-group.” These problems take two steps to answer. Here is an example of a fraction-of-a-group problem:

This morning \( \frac{2}{5} \) of Mrs. Raj’s 30 students rode the bus to school. How many students rode the bus?
Making a diagram for a fraction problem can help us understand the problem. We draw a rectangle to stand for the whole group of 30 students. The denominator of the fraction in the problem is five, so we divide the rectangle into fifths. Dividing 30 by 5, we find that there are 6 students in each fifth. We label the two fifths \( \frac{2}{5} \) that rode the bus. The rest \( \frac{3}{5} \) did not ride the bus.

We count the number of students in two fifths of the whole and find that 12 students rode the bus.

**Verify**

We can check the answer by adding the fractions. What is \( \frac{2}{5} + \frac{3}{5} \)?

**Example 1**

Nieve scored \( \frac{2}{3} \) of her team’s 36 points. How many points did she score?

We draw a rectangle to stand for the team’s 36 points. The denominator of the fraction in the problem is 3, so we divide the rectangle into thirds. One third of 36 is 12. We write “12 points” in each third of the rectangle. Since Nieve scored two thirds of the points, she scored 12 plus 12 points, or **24 points**.

**Justify**

Explain why the answer is correct.

**Example 2**

La’Donna’s dog ate \( \frac{1}{4} \) of a dozen dog bones. How many dog bones were left?

We draw a rectangle to stand for all 12 dog bones. We divide the rectangle into fourths. One fourth of 12 is 3, so we write “3 dog bones” in each of the four equal parts.
Example 3

A fifth grade basketball team scored 32 points in their first game of the season. The team scored \( \frac{5}{8} \) of those points in the second half of the game. Which diagram shows the number of points the team scored in the first half of the game?

Dividing 32 points into 8 equal parts means there are 4 points in each part. Since \( \frac{5}{8} \) of the points were scored in the second half, we know that \( \frac{3}{8} \) were scored in the first half. The diagram on the right matches the given information.

Lesson

Represent Illustrate and solve fraction problems a–c.

a. Two fifths of the 30 students in the class played in the band. How many students played in the band?

b. Susan practiced playing the trumpet for \( \frac{3}{4} \) of an hour. For how many minutes did Susan practice playing the trumpet?

c. Three fifths of the 30 students were girls. How many boys were there?

Written Practice Distributed and Integrated

1. Walking at a steady rate, Ebony walked 11 miles in 3 hours. Write a mixed number that shows how many miles she walked each hour.
For problems 2–4, write an equation and find the answer.

2. The theater had 625 seats. If 139 seats were empty, how many seats were filled?

3. This line segment is 4 centimeters long. How many millimeters long is it?

4. Seven thousand passengers arrived on 8 ships. If each ship carried an equal number of passengers, how many passengers were on each ship?

5. What year was two centuries before 1976?

6. Draw a diagram to illustrate and solve this problem:

Nguyen was voted “Most Valuable Player” for scoring \( \frac{2}{3} \) of her team’s 48 points. How many points did Nguyen score?

7. Compare: \( \frac{1}{4} \) of 60 \( \bigcirc \) \( \frac{1}{3} \) of 60

8. Round 256 to the nearest hundred.

9. Draw a rectangle. Shade all but two fifths of it. What percent of the rectangle is not shaded?

10. A late movie on television ended at 3 minutes before midnight. At what time did the movie end?

11. How many inches long is this nail?

12. \( 3 \frac{3}{7} + 2 + \frac{2}{7} \)

13. \( 2 \frac{2}{5} - 1 \)

14. \( 3 \frac{2}{3} - \frac{1}{3} \)

15. \( 6 \frac{5}{12} - \left( 4 + 1 \frac{4}{12} \right) \)

16. \( 1396 + 727 + 854 + 4685 \)

17. \( $20 - (\$15.37 - \$12) \)
18. \( 97 + w = 512 \)  
19. \( 938 \times 800 \)  
20. \( 54 \times 7 \times 60 \)  
21. \( 9n = 5445 \)  
22. \( 3205 \div 10 \)  
23. \( 4826 + 4826 + 4826 + 4826 \)  
24. A whole circle is divided into fifths. Each fifth is what percent of the whole circle?  
25. \( * \) Connect Which arrow could be pointing to 1375 on the number line below?  
26. A recipe to make 48 bran muffins requires 3 cups of bran. Gianna would like to halve the recipe and make only 24 muffins. Which quotient below represents the amount of bran Gianna should use to make 24 muffins?  
27. The word rectangle comes from the Latin terms for “right corner.” In what way is a rectangle a “right corner” polygon?  
28. Multiple Choice Which of these numbers is divisible by both 2 and 9?  
A. 234  
B. 456  
C. 567  
D. 245  
29. Some caterpillars are \( 1\frac{3}{4} \) inches long. Is \( 1\frac{3}{4} \) inches closer to 1 inch or 2 inches?  
30. Estimate Lake Superior is 350 miles long. Lake Huron is 206 miles long. What is a reasonable estimate of how much longer Lake Superior is than Lake Huron? Explain your answer.
• Simplifying Mixed Measures

Power Up

facts

Power Up F

count aloud

Count up and down by tens between 0 and 200. Count up and down by hundreds between 0 and 2000.

mental math

a. Number Sense: A score is 20. How many is two score? . . . three score? . . . four score?

b. Estimation: Round 757 to the nearest ten. Then subtract 400. What is the number?

c. Number Sense: $1\frac{1}{2} + \frac{1}{2}$

d. Number Sense: $1\frac{1}{2} - \frac{1}{2}$

e. Fractional Parts: How much money is half of $3$?

f. Percent: Aoliyah says she has 25% of $4.00 in her pocket. How much money does Aoliyah have?

g. Geometry: An octagon has how many more sides than a hexagon?

h. Calculation: $9 \times 8, - 2, \div 2, + 1, \div 4, + 1, \div 2$

problem solving

Choose an appropriate problem-solving strategy to solve this problem. Jiro wrote a division problem and then erased the two-digit dividend. He then gave it to Jenaya as a problem-solving exercise. Copy Jiro’s division problem and find the missing digits for Jenaya.

New Concept

In this lesson we will practice changing measures named with two units into measures named with one unit. In Examples 1–4, we will learn how to change these kinds of measurements.
**Example 1**

Jaylen is 5 feet 4 inches tall. How many inches tall is Jaylen?

Five feet 4 inches means “5 feet plus 4 inches.” Before we can add, we first change 5 feet to inches. Since 1 foot equals 12 inches, we multiply 5 by 12 inches.

\[ 5 \text{ feet} = 5 \times 12 \text{ inches} \]
\[ 5 \text{ feet} = 60 \text{ inches} \]

Now we add 60 inches and 4 inches.

\[ 60 \text{ inches} + 4 \text{ inches} = 64 \text{ inches} \]

Jaylen is 64 inches tall.

**Example 2**

Carla ran a quarter mile in 1 minute 15 seconds. What was her time in seconds?

One minute 15 seconds means “1 minute plus 15 seconds.” We first change 1 minute to seconds. Then we add.

\[ 1 \text{ minute} = 60 \text{ seconds} \]
\[ 60 \text{ seconds} + 15 \text{ seconds} = 75 \text{ seconds} \]

Carla ran a quarter mile in 75 seconds.

**Analyze** If the time on Carla’s digital watch read 3:05:06 when she began running, what time was it when she crossed the finish line?

**Example 3**

The melon weighed 3 pounds 8 ounces. How many ounces did the melon weigh?

Three pounds 8 ounces means “3 pounds plus 8 ounces.” We change pounds to ounces first. One pound equals 16 ounces.

\[ 3 \text{ pounds} = 3 \times 16 \text{ ounces} \]
\[ 3 \text{ pounds} = 48 \text{ ounces} \]

Now we add.

\[ 48 \text{ ounces} + 8 \text{ ounces} = 56 \text{ ounces} \]

The melon weighed 56 ounces.
In the refrigerator, there was a full one-gallon container of milk and a full one-quart container of milk. Convert 1 gallon to quarts and find the sum of the capacities of the containers.

A quart is one quarter \( \frac{1}{4} \) of a gallon, so one gallon is 4 quarts.

One gallon plus one quart is 5 quarts.

**Activity**

**Simplifying Height Measurements**

Material needed:
- yardstick

Choose a height in the classroom, such as the height of your desk, a windowsill, or a bookshelf. Measure the height you chose in inches only. Then measure the height you chose in feet and inches. Show that the two measurements represent the same distance.

**Lesson Practice**

Use one unit to name each measure:

- \( \text{a. } 6 \text{ feet } 2 \text{ inches } = \_\_\_ \text{ inches} \)
- \( \text{b. } 3 \text{ minutes } 2 \text{ seconds } = \_\_\_ \text{ seconds} \)
- \( \text{c. } 2 \text{ hours } 30 \text{ minutes } = \_\_\_ \text{ minutes} \)
- \( \text{d. } 2 \text{ pounds } 12 \text{ ounces } = \_\_\_ \text{ ounces} \)

**Written Practice**

**Distributed and Integrated**

**Formulate** For problems 1–3, write and solve an equation to find the answer.

1. There were 36 students on one bus, 29 on another bus, and 73 on the third bus. Altogether, how many students were on the three buses? Explain how you know your answer is reasonable.

2. Anita’s grandfather has lived for seven decades. Seven decades is how many years?

3. Anita is 12 years old. Her grandmother is 68 years old. Anita’s grandmother is how many years older than Anita?
*4. When Gabriel turned 12 years old, he was 5 feet 6 inches tall. How many inches is 5 feet 6 inches?  

*5. **Multiple Choice**  The 7 in 374,021 means which of the following?  
A 7  
B 70  
C 700  
D 70,000  

6. From March 1 of one year to May 1 of the next year is how many months?  

*7. **Represent**  Draw a rectangle. Shade three eighths of it. What percent of the rectangle is shaded?  

*8. Use a ruler to find the length of this line segment in inches:  

*9. 4 + 3\(\frac{3}{4}\)  
10. \(3\frac{3}{5} + 1\frac{1}{5}\)  
11. \(2\frac{3}{8} + 2\frac{2}{8}\)  
*12. \(5\frac{1}{3} - \left(5\frac{1}{3} - \frac{1}{3}\right)\)  

*13. \(2\frac{1}{2} - \frac{1}{2}\)  
*14. \(3\frac{5}{9} - 1\frac{1}{9}\)  
15. \$48,748 - \$37,145  
16. \$63,142 - \$17,936  

17. \(\frac{5.63}{700}\)  
18. \(\frac{4729}{8}\)  
19. \(\frac{9006}{80}\)  
20. \(\frac{3456}{8}\)  

21. \(\frac{1836}{9}\)  
22. \(\frac{1405}{7}\)  
*23. \((20 \times 25) + (5 \times 25)\)  

*24. In a packet of 50 flower seeds, \(\frac{2}{5}\) of the seeds are daisies. If Victor plants the seeds in a garden and all of them sprout and grow, how many daisy plants can Victor expect to find in his garden?  

*25. Each school day Amy’s alarm clock rings at 6:45 a.m. This morning, Amy fell back to sleep after turning off her alarm and woke up again at the time shown on the clock. How many fewer minutes did Amy have this morning to get ready for school?
26. **Explain** If 115 students ride on 3 school buses, is it possible for the same number of students to ride on each bus? Explain why or why not.

27. **Multiple Choice** Which of these numbers is divisible by 6 and by 5?
   \[ A \ 576 \quad B \ 765 \quad C \ 6057 \quad D \ 7650 \]

28. Compare: \( \frac{1}{5} \) of 10 \( \bigcirc \) 10 ÷ 5

29. Some grasshoppers grow to be \( 2 \frac{1}{2} \) inches long. Is \( 2 \frac{1}{2} \) inches closer to 2 inches or to 3 inches?

30. Although the amount of food a bottlenose dolphin eats in a day can vary from one day to the next, the table below shows the average amount of food a bottlenose dolphin eats.

   **The Bottlenose Dolphin**

<table>
<thead>
<tr>
<th>Number of Days</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average Amount of Food Eaten (in pounds)</td>
<td>23</td>
<td>46</td>
<td>69</td>
<td>92</td>
</tr>
</tbody>
</table>

   a. **Generalize** Write a rule that describes how to find the average amount of food eaten in pounds for any number of days.

   b. **Predict** What is a reasonable estimate of the amount of food a bottlenose dolphin can be expected to eat in 1 week? Explain why your estimate is reasonable.

---

**Real-World Connection**

Shelby is running in a relay race on a team of four runners. She ran her lap in 1 minute 23 seconds. Her team’s total time was 5 minutes 9 seconds.

   a. Change both of these times to seconds.

   b. How much of the team’s total time was not run by Shelby? Give your answer in minutes and seconds.
LESSON 48

• Reading and Writing Whole Numbers in Expanded Notation

Power Up

facts
Count by 3s from 3 to 36.

mental math
a. **Geometry:** A pentagon has 5 sides. If each side of a pentagon is 45 millimeters long, what is the distance around the pentagon?

b. **Time:** Eli needs to leave his house 25 minutes before 2:45 p.m. What time does he need to leave?

c. **Measurement:** Perry’s height is 5 feet 9 inches. What is this height in inches?

d. **Number Sense:** Jacinta could type 90 words in one minute. At that rate, how many words could she type in 6 minutes?

e. **Fractional Parts:** $\frac{1}{2}$ of 17

f. **Number Sense:** $\frac{3}{10} + \frac{7}{10}$

g. **Number Sense:** $\frac{1}{4} + \frac{3}{4}$

h. **Calculation:** $8 \times 7, + 4, \div 2, + 6, \div 4, + 1, \div 2$

problem solving
Choose an appropriate problem-solving strategy to solve this problem. Six dots can make a triangular pattern with three dots on each side. Ten dots can make a triangular pattern with four dots on each side. How many dots are in a triangular pattern that has seven dots on each side?
**Focus Strategies:** Draw a Picture; Find a Pattern; Write an Equation

**Understand** We are shown two triangular patterns of dots. The triangle with six dots has three dots on each side. The triangle with ten dots has four dots on each side. We are asked to find the number of dots in a triangular pattern that has seven dots on each side.

**Plan** We can draw a picture of the triangular pattern and then count the dots.

**Solve** To draw the pattern, we can start at the top of the triangle by drawing 1 dot. Then we can place a “row” of 2 dots beneath the first dot. Then we can place a row of 3 dots beneath the 2 dots. We continue until we place a row of 7 dots at the bottom. We count 28 dots altogether.

**Check** We know that our answer is reasonable because we can count the dots in our picture. We could also find a pattern and write an equation for the number of dots. Each row of dots in each triangle contains one more dot than the row above it. This means the triangle with three dots on each side has \(1 + 2 + 3 = 6\) dots. The triangle with four dots on each side has \(1 + 2 + 3 + 4 = 10\) dots. A triangular pattern with seven dots on each side has \(1 + 2 + 3 + 4 + 5 + 6 + 7 = 28\) dots.

**New Concept**

One way to name numbers is to name the place value of each digit. The number 3256 could be named

\[
3 \text{ thousands} + 2 \text{ hundreds} + 5 \text{ tens} + 6 \text{ ones}
\]

We could use numbers instead of words to rewrite this as

\[
(3 \times 1000) + (2 \times 100) + (5 \times 10) + (6 \times 1)
\]

This method of naming numbers is called **expanded notation**. When we write a number in expanded notation, we write a digit times its place value, plus the next digit times its place value, and so on.
Example 1
Write the number 5600 in expanded notation.
The number 5600 is 5 thousands + 6 hundreds + 0 tens + 0 ones.
We write 5 times its place value, plus 6 times its place value. Since there are no tens or ones, we write only
\[(5 \times 1000) + (6 \times 100)\]

Example 2
Write 750,000 in expanded notation.
The number 750,000 is 7 hundred thousands and 5 ten thousands.
We write 7 times its place value plus 5 times its place value.
\[(7 \times 100,000) + (5 \times 10,000)\]

Analyze What is 4,000,000 written in expanded form?

Example 3
Write the standard form for \((3 \times 100) + (2 \times 1)\).
Standard form means “the usual way of writing numbers.” We will write the number that has a 3 in the hundreds place and a 2 in the ones place.

\[
\begin{array}{ccc}
100s & 10s & 1s \\
3 & 0 & 2 \\
\end{array}
\]

Note that we use a zero to hold the tens place. The standard form is 302.

Analyze What is the expanded form of the whole number 3?

Lesson Practice
Represent Write in expanded notation:

a. 56
b. 5280
c. 250,000

Represent Write in standard form:
d. \((6 \times 1000) + (4 \times 10)\)
e. \((5 \times 100) + (7 \times 10)\)
f. \((8 \times 10,000) + (4 \times 1000)\)
g. \((9 \times 100,000) + (3 \times 10,000)\)
*1. Represent  Draw a square. Make each side \(1\frac{1}{2}\) inches long.

Formulate  For problems 2–4, write an equation and find the answer.

*2. Aizza is 6 years older than Juno. If Aizza is 21, then how old is Juno?

3. It takes a tour guide 50 minutes to lead a group through three different rooms of a museum. If the tour guide spends an equal amount of time in each room, which quotient below shows how much time she spends in each room?

\[
\begin{array}{c@{\hspace{1cm}}c}
16 \frac{2}{3} & 16 \text{ R } 2 \\
3)50 & 3)50 \\
-3 & -3 \\
20 & 20 \\
-18 & -18 \\
2 & 2
\end{array}
\]

4. Eight dozen eggs is how many eggs?

*5. Multiple Choice  The 6 in 356,287 represents which of the following numbers?

\[\begin{array}{c}
\text{A} & 6 \\
\text{B} & 356 \\
\text{C} & 6000 \\
\text{D} & 6287
\end{array}\]

6. The new pencil was 19 centimeters long. How many millimeters long was the pencil?

*7. Represent  Draw a circle. Shade one sixth of it. What percent of the circle is shaded?

8. Round 287 to the nearest ten.

9. Represent  Write the standard form for \((5 \times 100) + (2 \times 1)\).

10. Represent  Write the number 47,000 in expanded notation.
11. \( \begin{array}{ccc} 98,572 \\ 42,156 \\ 37,428 \\ + 16,984 \end{array} \)

12. \( W \)

\[ \begin{array}{c} - 32,436 \\ \hline 19,724 \end{array} \]

13. \( \begin{array}{c} 10,000 \\ - y \end{array} \)

\[ \frac{1,746}{1,746} \]

14. \( $34.78 \)

\[ \times \frac{6}{6} \]

15. \( 6549 \)

\[ \times \frac{60}{60} \]

16. \( 8037 \)

\[ \times \frac{90}{90} \]

17. \( 3647 \div 6 \)

18. \( 5408 \div 9 \)

19. \( 10W = 1000 \)

20. \( \frac{3}{3} + \left( \frac{4}{3} - 2 \right) \)

21. \( 6 \times 800 = 6 \times 8 \times h \)

22. \( $10 - ($6 + $1.47 + $0.93) \)

23. \( (20 \times 62) + (3 \times 62) \)

24. Analyze How many years is one fourth of a century? You may draw a diagram to answer the question.

25. What time is 1 minute before midnight?

26. Connect To what number is the arrow pointing?
27. The average fuel economy of an automobile is shown in this table:

<table>
<thead>
<tr>
<th>Amount of Fuel Used (in gallons)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Distance Traveled (in miles)</td>
<td>28</td>
<td>56</td>
<td>84</td>
<td>112</td>
</tr>
</tbody>
</table>

a. Name the next three amounts in the pattern.

b. **Predict** Suppose that the fuel tank of the automobile has a capacity of 10 gallons. What is a reasonable estimate of the distance the car can travel using one tank of fuel?

28. If a dollar's worth of dimes is divided into two equal groups, how many dimes will be in each group?

*29. A stop sign is a traffic sign that is shaped like an octagon. How many sides does a stop sign have?

30. **Estimate** Workers in Italy receive an average of 42 vacation days per year. Workers in the United States receive an average of 13 vacation days per year. Explain how to use rounding to estimate how many more vacation days workers in Italy receive each year.

According to the 2005 U.S. Census estimates, Austin, Texas, had a population of 690,252 people.

a. Write this number in expanded form.

b. Write this number using words.
• Solving Multiple-Step Word Problems

**Power Up**

**facts**

**count aloud**

Count up and down by 25s between 250 and 500.

**mental math**

a. **Estimation:** The bottle of water weighs 521 grams. Round this weight to the nearest hundred grams.

b. **Money:** The price of the refrigerator was $740. The delivery fee was $60. What was the total cost?

c. **Number Sense:** \( \frac{4}{10} + \frac{5}{10} \)

d. **Number Sense:** \( \frac{9}{10} + \frac{1}{10} \)

e. **Percent:** 50% of 10

f. **Percent:** 10% of 10

g. **Fractional Parts:** \( \frac{1}{3} \) of 12 inches is how many inches?

h. **Calculation:** \( 12 \div 2, + 2, \div 2, + 2, \div 2, + 2 \)

**problem solving**

Choose an appropriate problem-solving strategy to solve this problem. Enrique rolled two dot cubes. The total was 7. Copy this table and write all the ways Enrique could have rolled a total of 7.

<table>
<thead>
<tr>
<th>First Cube</th>
<th>Second Cube</th>
</tr>
</thead>
</table>

**New Concept**

Many mathematical problems take more than one step to solve. We have solved several kinds of two-step problems. For example, it takes two steps to solve \( 10 - (6 - 3) \), and we used two steps to convert 6 feet 2 inches to inches. We have also illustrated and solved fraction problems that take two steps.
The first step of a multiple-step word problem often involves finding a number that does not appear in the problem. We then use the number we found to help us solve the problem.

Example 1

Huang is 5 years older than Robert. Robert is 3 years older than Sally. Sally is 15 years old. How old is Huang?

This two-step problem is actually two “larger-smaller-difference” problems put together into one problem. We are asked to find Huang’s age, but we cannot find his age in one step. We must first find Robert’s age, and we are given enough information to find it.

**Goal:** Find Huang’s age.

**First step:** Find Robert’s age.

We will draw a pair of rectangles for each comparison.

Drawing the rectangles helps us see that Robert is 18 years old. We write “18” in both rectangles that stand for Robert’s age.

Since Huang is 5 years older than Robert, he is 18 years plus 5 years old, which is **23 years old**.
Example 2

Liam earns $16 for every hour he works. If he works for more than 8 hours on a given day, he earns $24 for each additional hour. Yesterday Liam worked for 10 hours. What was his income for the day?

Liam works at two different pay rates.

**Step 1:** Find his pay at $16 per hour. We multiply 8 hours times $16 per hour and find that Liam earned $128.

**Step 2:** Find his pay at $24 per hour. Since he worked for 10 hours, we know that Liam earned $24 for 2 of those hours. We multiply 2 hours times $24 per hour and find that Liam earned $48.

**Step 3:** Find his total pay. We add the amount Liam earned in the first 8 hours ($128) to the amount he earned in the next two hours ($48) and find that he earned a total of $176 for 10 hours of work.

Example 3

Last night, Kiara studied for twice as long as Tempest, and Alondra studied for 10 minutes more than Kiara. Tempest studied for 40 minutes.

Which expression can be used to find the number of minutes Alondra studied?

A  \((40 \times 2) + 10\)  
B  \((40 \times 2) - 10\)  
C  \((40 \div 2) - 10\)  
D  \((40 \div 2) + 10\)

Alondra’s study time is compared to Kiara’s, so the first step is to find how long Kiara studied. Since Kiara studied twice as long as Tempest, and Tempest studied for 40 minutes, we can represent how long Kiara studied this way: \((40 \times 2)\). Alondra studied for 10 minutes more than Kiara, so Alondra’s study time is represented by choice A.

Lesson Practice

Solve each two-step problem:

a. The bus could carry 80 students. The 32 students from Room 8 and the 29 students from Room 12 got on the bus. How many more students can the bus carry? (Goal: Find how many more students will fit. First step: Find how many students are already on the bus.)
b. Mariabella collected 37 aluminum cans for school, and her brother collected 21. They decided to divide the cans evenly, so they put the cans into one big pile and then made two equal piles. How many cans were in each pile? (Goal: Find how many cans are in each pile. First step: Find how many cans there are in all.)

Written Practice

*1. (31, 32) **Represent** Draw a quadrilateral that has one pair of parallel sides and one pair of sides that are not parallel.

*2. (49) **Analyze** Kafele is 10 years older than Hakim. Hakim is 5 years older than Ciante. Ciante is 15 years old. How old is Kafele? (Goal: Find Kafele’s age. First step: Find Hakim’s age.)

3. Adriana’s age is \( \frac{1}{3} \) of her dad’s age. If her dad is 36 years old, how old is Adriana?

4. (46) **Represent** Draw a diagram to illustrate and solve this problem:

   On Monday, \( \frac{5}{6} \) of the 24 students in a fifth grade class were in school. The remainder of the students were absent. How many students were absent that day?

*5. (7) **Multiple Choice** The 7 in 754,238 represents which of the following numbers?

   A 700,000   B 700   C 7   D 754

6. (48) **Represent** Write the standard form for \( (5 \times 100) + (6 \times 1) \).

7. Zuri and Kya will earn $5 for watering the garden. If they share the money equally, how much money will each person receive?

8. Round 234 to the nearest ten.

9. (7) **Represent** Use digits to write twenty-five thousand, three hundred.
10. **Represent** Draw a circle. Shade five sixths of it. What percent of the circle is not shaded?

11. \( \frac{5}{8} + 6 + \frac{3}{8} \)

12. \( \frac{8}{6} - \left( \frac{5}{6} - 3 \right) \)

13. \( 342 + 5874 + 63 + 285 + 8 + 96 + 87 \)

14. \( $42.01 - $20.14 \)

15. \( 1000 - m = 1 \)

16. \( 800 \times 50 \)

17. \( 30 \times 8 \times 25 \)

18. \( 1205 \div 6 \)

19. \( $76.32 \div 8 \)

20. \( $20 - ($12 + $4.76 + $2.89 + $0.34) \)

21. \( (20 \times 35) + (5 \times 35) \)

22. **Represent** Use words to name the number 150,000.

23. **Represent** Write 150,000 in expanded notation.

24. **Conclude** The average number of calories used by a hiker are shown in this table:

<table>
<thead>
<tr>
<th>Time Spent Hiking (in minutes)</th>
<th>30</th>
<th>60</th>
<th>90</th>
<th>120</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Calories</td>
<td>185</td>
<td>370</td>
<td>555</td>
<td>740</td>
</tr>
</tbody>
</table>

Name the next three numbers of calories in the pattern.

25. Which place does the zero hold in 203,456?

26. **Connect** To what number is the arrow pointing?

27. Omar was born on December 1, 1998. Omar’s mom was born on January 18, 1965. How old was Omar’s mom when Omar was born?
28. Think of an odd number. Multiply it by 5. What is the last digit of the product?

29. **Multiple Choice** Which of these angles appears to be an obtuse angle?
   
   A  
   B  
   C  
   D  

30. **Estimate** The highest and lowest temperatures ever recorded in Phoenix, Arizona, are shown on the thermometers below. When compared to the highest temperature, about how many degrees colder is the lowest temperature? Explain why your estimate is reasonable.

   ![Thermometers](image)

   Alfonzo is 8 years older than his brother. His brother is 4 years older than their sister. Their sister is 11 years old. Use a problem-solving strategy to find the age of Alfonzo and his brother. Which problem-solving strategy did you use? Explain how you used the strategy to solve.
• Finding an Average

**Power Up**

- **facts**
- **count aloud**
- **mental math**

**Power Up F**

- Count up and down by 3s between 3 and 36.

**a. Estimation:** Estimate the sum of $89 and $58.

**b. Measurement:** 870 grams plus 130 grams is one kilogram. How many grams is one kilogram?

**c. Number Sense:** $1\frac{1}{2} + 1\frac{1}{2}$

**d. Number Sense:** $3\frac{1}{3} - 2$

**e. Fractional Parts:** $\frac{1}{3}$ of the 15 coins are dimes. What is the total value of the dimes?

**f. Percent:** 25% of 16 ounces is how many ounces?

**g. Time:** It takes 8 seconds to print one page. How many seconds will it take to print 34 pages? *(Think: $8 \times 34$)*

**h. Calculation:** $6 \times 5, + 3, \div 3, + 4, \div 3, + 1, \div 3$

**problem solving**

Choose an appropriate problem-solving strategy to solve this problem. The numbers 3, 6, and 10 may be called **triangular numbers.** This is because 3 objects, 6 objects, and 10 objects can each be arranged in a triangular pattern. What are the next three triangular numbers?

**New Concept**

Below we show two stacks of nickels. In one stack there are 5 nickels, and in the other stack there are 9 nickels. If some nickels were moved from the taller stack to the shorter stack so that the stacks were even, how many nickels would be in each stack?
One way to answer this question is to first **find the total** number of nickels and then divide the total into two **equal groups**. Since there are 5 nickels in one stack and 9 nickels in the other stack, there are 14 nickels in all. Dividing 14 nickels into 2 equal groups, we find that there would be 7 nickels in each stack.

When we even up the number of members in groups, we are finding the **average** number of members per group. Finding an average is a two-step process.

**Step 1:** Combine to find the total.

**Step 2:** Separate the total into equal groups.

### Example 1

If water is poured from glass to glass until the amount of water in each glass is the same, how many ounces of water will be in each glass?

The **total amount** of water will be **divided equally** among the three glasses. Finding the total amount of water is a problem about combining. It has a “some and some more” (addition) pattern. We add and find that the total amount of water is 18 ounces.

\[
\begin{align*}
4 \text{ ounces} & \quad 7 \text{ ounces} \quad 7 \text{ ounces} \\
+ \quad 7 \text{ ounces} & \\
\hline
18 \text{ ounces} & 
\end{align*}
\]

Finding the amount for each glass is an “equal groups” problem. “Equal groups” problems have multiplication patterns. We divide 18 ounces by 3 and find that there will be **6 ounces of water** in each glass.

\[
\begin{align*}
n \text{ ounces in each glass} & \quad 6 \\
\times 3 \text{ glasses} & \quad 3)18 \\
18 \text{ ounces in all 3 glasses} & 
\end{align*}
\]

**Analyze** If we added a fourth glass with 6 ounces of water in it, would the average amount of water in each glass change? Why or why not?
Example 2

Brad’s sister timed him as he swam laps. Brad’s lap times in seconds are 80, 85, 90, 85, and 90. What is the average of Brad’s lap times?

Finding an average takes two steps. The first step is to find the total. To do this, we add Brad’s times.

\[80 + 85 + 90 + 85 + 90 = 430\]

The second step is to separate the total into equal groups. Brad swam five laps, so we divide the total into five equal parts.

\[430 \div 5 = 86\]

We find that Brad’s average time is 86. Notice that although none of Brad’s times were 86, the sum of the five times, 430, is the same as if he swam every lap in 86 seconds.

Lesson Practice

Solve each two-step problem by combining and then forming equal groups:

a. The number of players on the four squads was 5, 6, 9, and 8. If the squads were changed so that the same number of players were on each squad, how many players would each squad have?

b. When the class lined up, there were 11 students in one line and 17 students in the other line. If the lines were rearranged to have the same number of students, how many students would be in each line?

c. This picture shows three stacks of books. If the stacks were equal, how many books would be in each stack?

d. Here are Shauna’s game points:

8, 9, 7, 9, 8, 10, 6, 7

What is the average of Shauna’s game points?
*1. **Represent**  Draw a quadrilateral so that the sides that intersect are perpendicular.

2. **Represent**  Kimberly is 5 years older than Loh. Miguel is 2 years older than Loh. Miguel is 13 years old. How old is Kimberly? Draw a pair of rectangles for each comparison.

*3. **Analyze**  If water is poured from glass to glass until the amount of water in each glass is the same, how many ounces of water will be in each glass? (First use an addition pattern to find the total amount of water. Then use a multiplication pattern to divide the total equally.)

*4. **Represent**  Draw a diagram to illustrate and solve this problem:

   How many minutes is \( \frac{3}{5} \) of an hour?

*5. How many minutes is 2 hours 15 minutes?

6. Four hundred years is how many centuries?

*7. **Represent**  Use digits to write fifty-four thousand, nine hundred nineteen.

*8. **Represent**  Draw a rectangle. Shade seven eighths of it. What percent of the rectangle is not shaded?

9. There were 15 children in one line and 11 children in another line. After some children moved from the longer line to the shorter line, there were the same number of children in each line. How many children were there in each line?

10. 342 + 67 + 918 + 897 + 42
11. \(53.87 - 27.59\)  
\((13)\)

12. \(34.28 \times 60\)  
\((29)\)

13. \(7 \times 57 \times 10\)  
\((18, 29)\)

14. \((4 + 7 + 7) \div 3\)  
\((24)\)

15. \((5 + 6 + 9 + 8) \div 4\)  
\((24)\)

16. \(4206 \div 7\)  
\((34)\)

17. \(60.24 \div 6\)  
\((34)\)

18. \(1000 \div 9\)  
\((26)\)

19. \(7 \times 57 \times 10\)  
\(3990\)

20. \(6d = 180\)  
\((26, 34)\)

21. \(\frac{9}{10} - \left(\frac{7}{10} - \frac{5}{10}\right)\)  
\((24, 41)\)

22. \((10 \times 43) + (2 \times 43)\)  
\((24, 29)\)

23. What month is 10 months after July?

24. **Multiple Choice** When we count by hundreds, we find that 1236 is closest to which of the following numbers?

   - A 1100
   - B 1200
   - C 1300
   - D 1000

25. Use the information below to answer parts a–c. You may draw a map.

   From Safara’s house, Arcadia Park is 4 miles north, Legg Lake is 5 miles south, the ocean is 32 miles west, and the mountain cabin is 98 miles east.

   a. Safara’s family went to the ocean one Saturday. They left home at 9 a.m. and returned home at 4 p.m. How long were they gone?

   b. How far is it from Arcadia Park to Legg Lake?

   c. How far did they travel when they went to the mountain cabin and then back home?
26. Dexter kept a record of his bike rides. On average, how far did Dexter ride each day?

<table>
<thead>
<tr>
<th>Day</th>
<th>Distance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Monday</td>
<td>5 miles to park</td>
</tr>
<tr>
<td>Tuesday</td>
<td>8 miles to river</td>
</tr>
<tr>
<td>Wednesday</td>
<td>8 miles to river</td>
</tr>
<tr>
<td>Thursday</td>
<td>6 miles to bridge</td>
</tr>
<tr>
<td>Friday</td>
<td>8 miles to river</td>
</tr>
</tbody>
</table>

27. Seven hundred megabytes of data can be stored on a compact disc. Write an equation that shows the amount of data that can be stored on five compact discs. Use a to represent the amount of data.

28. Predict A sequence of letters of the alphabet is shown below:

   c, f, i, l, o

   What letter represents the eighth term of the sequence?

29. The length of the North Canadian River in New Mexico and Oklahoma is 200 miles longer than twice the length of the Rock River in Illinois and Wisconsin. The Rock River is 300 miles long. What is the length of the North Canadian River?

30. Estimate The Giessbach waterfall in Switzerland has a height of 984 feet. The Tully waterfall in Australia has a height of 885 feet. Explain how to estimate the height difference of the waterfalls.

In his first four games, Neil had bowling scores of 112, 126, 98, and 118. What score must Neil bowl in his fifth game to have an average of 120 for all five games?
Focus on

• Organizing and Analyzing Data

Your teacher collects and records information about your class, including attendance, homework, and test scores. Your school gathers information about class sizes, lunch counts, and supplies ordered. Sports teams compile information about wins and losses, points scored, and points allowed. Researchers testing new medications keep careful records of subjects who respond well, of subjects who do not respond, and of subjects who experience side effects.

These types of gathered information are called data, and the study of data is called statistics. People who work in the field of statistics collect, organize, analyze, and display data.

Newly gathered data are often unorganized. To be useful, these “raw” data must first be organized. In this investigation, we will practice two ways of organizing data by making and using frequency tables and line plots. A frequency table displays the number of times an event or outcome occurs. Then we will practice analyzing data to solve problems.

Frequency Tables

Mr. Sottong gave his 25 students a choice of 6 activities. Each student completed the activity of his or her choice. A student could have completed activity 1, 2, 3, 4, 5, or 6. Here are the activities that Mr. Sottong’s students chose to complete:

4, 3, 3, 4, 2, 5, 6, 1, 3, 4, 5, 2, 2, 6, 3, 4, 3, 2, 4, 5, 3, 5, 5, 6

Mr. Sottong decides to organize the data using tally marks in a frequency table that shows how many students completed each possible activity. He lists the possible activities and then tallies the number of students who chose each activity.
After tallying, Mr. Sottong counts the number of marks for each activity to get the frequency of each score. He lists these frequencies in a third column, as shown below:

<table>
<thead>
<tr>
<th>Activity</th>
<th>Tally</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

1. Twenty children were asked how many brothers and sisters they each had. The responses were these numbers:
   2, 3, 0, 1, 1, 3, 0, 4, 1, 2,
   0, 1, 1, 2, 2, 3, 0, 2, 1, 1

Represent Make a frequency table that shows the numbers of brothers and sisters.

Suppose the ABC Market offers 22 turkeys for sale. The weights of the turkeys in pounds are given below:

11, 18, 21, 23, 16, 20, 22, 14, 16, 20, 17,
19, 13, 14, 22, 19, 22, 18, 20, 12, 25, 23

Sometimes we group our data in equal intervals. On the next page, the frequency table for these data uses intervals of 4 pounds, starting with the interval 10–13 lb. (This interval includes the weights 10, 11, 12, and 13 pounds.)
2. **Represent** Use the raw data about the turkey weights at ABC Market to make a new frequency table. Instead of the weight intervals shown in the table above, use the intervals 11–15 lb, 16–20 lb, and 21–25 lb.

3. The annual rainfall in a certain Texas city for each of the last fifteen years is recorded below. Each of these data is in inches:

\[17, 24, 32, 27, 18, 30, 22, 18, 24, 31, 32, 26, 18, 19, 22\]

**Represent** Make a frequency table for the data, using 5-inch intervals. Begin with the interval 15–19 in.

### Line Plots

If we want to see each data point, we can create a **line plot**. We draw a number line that includes the highest and lowest values we collected. Then we place an X above each number for each data point that corresponds to that number. Above some numbers, there might be a stack of Xs. Above other numbers, there might be none.

Suppose Jean-Paul recorded the ages of the first 20 people who went down a certain water slide after the water park opened at 9 a.m. Their ages were

\[11, 9, 8, 11, 12, 15, 13, 12, 17, 12, 12, 22, 13, 11, 21, 9, 16, 12, 13, 9\]

To make a line plot of this information, we first draw a number line. If we begin our number line at 5 and end it at 25, all of the data points can be included. Now we place an X on the number line for each data point. Since there are 3 data points that have value 9, we stack 3 Xs above 9 on the number line.
A **cluster** is a group of data points that are very close together. We see that the data points 11, 12, and 13 form a cluster. There can be more than one cluster in a data set. An **outlier** is a data point that is distant from the majority of data points. In this set, the data points 21 and 22 could be considered outliers.

**Interpret** Refer to the line plot on the previous page to answer problems 4–6.

4. Which age was recorded most frequently?
5. How many people who went down the slide were over 15?
6. Which ages were recorded exactly three times?

On a line plot, we can easily spot the **mode**, which is the most frequently occurring number in a set of data. The mode of the ages is 12.

**Analyze** How can we identify the mode by looking at the line plot?

We can also easily find the **range** on a line plot. The range is the difference between the greatest number and the least number in a set of data. On the line plot on the previous page, the greatest age is 22 and the least is 8, so the range is $22 - 8 = 14$ years.

The **median** of a set of data is the middle number. Half of the numbers are less than the median and half of the numbers are greater than the median. To find the median, we arrange the numbers in order and look for the middle number. A line plot arranges the numbers in a set of data in order, so we can find the median by counting the Xs. Since our line plot has 20 Xs, the number that has 10 Xs on both sides of it is the median.

**Explain** Describe how to find the median age in the set of data shown on the line plot on the previous page.

We can also find the median by listing the ages in order and counting halfway up or down.

$$8, 9, 9, 9, 11, 11, 11, 12, 12, 12, 12, 12, 12, 13, 13, 13, 15, 16, 17, 21, 22$$

**Interpret** At the top of the next page, we show another line plot. Refer to this line plot to answer problems 7–11.

*The Lizard Emporium pet store tracks the life span of the iguanas it sells. The data obtained for 21 iguanas are displayed in the line plot on the next page.*
7. What was the most frequent life span (mode) of the iguanas that were tracked?

8. How many iguanas were tracked?

9. How many iguanas lived between 17 and 25 years?

10. What is the range of the life spans?

11. What is the median life span of the iguanas?

12. **Represent** The teacher recorded the number of papers she graded each day for 15 days. Make a line plot for the following numbers of papers graded each day:

   19, 18, 17, 15, 18, 20, 14, 17,
   19, 11, 18, 17, 16, 18, 16

   **Interpret** Refer to your line plot to answer problems 13–15.

13. Which number is an outlier?

14. Which number occurs most often (mode)?

15. What is the median for this set of data?

**Analyzing Data**

Part of the process of solving problems is finding the information needed to solve the problem. We can find information in graphs, tables, books, on the Internet, and in other places. Besides locating the information, we often need to sort through the information to find the facts needed to solve the problem.

**Example 1**

The **pictograph** on the next page shows the number of dinners served at a restaurant each day it was open last week. Refer to the graph to answer problems a and b.

a. How many dinners were served on the day that the most dinners were served?

b. What is the mode for the set of data displayed on the “Dinners Served” graph?
Refer to the information in Example 1 to answer problems 16 and 17.

16. What is the range of the number of dinners served?
17. What is the median of the number of dinners served?

Example 2

Each student in the class was asked to name his or her favorite school lunch from a choice of four menus. The results are recorded in this bar graph:

According to the bar graph, how many students chose pizza as their favorite school lunch?

The bar for pizza ends halfway between the line for 8 and the line for 10. Halfway between 8 and 10 is 9. Thus, pizza was the favorite food of 9 students.

Refer to the bar graph in Example 2 to answer problems 18 and 19.

18. What is the total number of students represented by the four bars in the graph?
19. Is it true that the number of students who named pizza as their favorite was twice the number who named spaghetti? How do you know?

**Analyse** Can we find the median of the information in the bar graph? Explain.

**Example 3**

In the line graph at right, Sharon plotted the altitude of a model airplane during a 70-second flight.

**For how many seconds was the plane descending before it landed?**

According to the graph, the plane reached its maximum altitude 40 seconds into the flight and landed 70 seconds after takeoff. It descended throughout the last **30 seconds** of the flight.

Refer to the information in Example 3 to answer problems 20 and 21.

20. What was the maximum altitude reached by the model airplane?

21. For how many seconds during the flight did the plane fly at a constant altitude?

**Activity**

Graphing

Material needed:
- Lesson Activity 32

Make a Pictograph

22. Sonora went to the Cockrell Butterfly Museum and saw many different species of butterflies. Sonora used a butterfly booklet to help her identify the different types of butterflies that she saw in the museum. The table at right shows the number of times that Sonora saw a certain species of butterfly. Use the table to make a pictograph showing the data Sonora collected.

<table>
<thead>
<tr>
<th>Butterflies Identified</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>The Julia</td>
<td>X</td>
</tr>
<tr>
<td>Rice Paper Butterfly</td>
<td>X</td>
</tr>
<tr>
<td>Blue Morpho</td>
<td>X</td>
</tr>
<tr>
<td>Green Triangle</td>
<td>X</td>
</tr>
<tr>
<td>Owl Butterfly</td>
<td>X</td>
</tr>
<tr>
<td>Red Peacock</td>
<td>X</td>
</tr>
</tbody>
</table>

(EIGHT IN FEET)
Using your pictograph, solve the following problems:

a. What would be the most reasonable number for each symbol to represent? Explain why.

b. What would half of a symbol represent?

c. After looking at the pictograph, which type of butterfly do you think the museum has the most of? Explain your answer.

d. How many owl butterflies and rice paper butterflies did Sonora see altogether?

Make a Bar Graph

23. Ask your classmates what type of movie they like to watch. Use the categories listed in the table for your survey. Make a bar graph to display the collected data. Use the table to make a bar graph showing the data you have collected.

Using your bar graph, solve the following problems:

a. What scale is used on your bar graph? Explain why you chose this scale.

b. Describe what your bar graph shows.

c. What is the total number of students represented in the bar graph?

Make a Pictograph or a Bar Graph

24. A pet store keeps track of the types of birds that are sold each month. The list shows the number of birds that were sold in April. Display the data using either a pictograph or a bar graph.

Using your pictograph, solve the following problems:

a. What would be the most reasonable number for each symbol to represent? Explain why.

b. What would half of a symbol represent?

c. After looking at the pictograph, which type of butterfly do you think the museum has the most of? Explain your answer.

d. How many owl butterflies and rice paper butterflies did Sonora see altogether?