• Multiplying Decimal Numbers by 10, by 100, and by 1000

Power Up

 facts
 mental math
 Power Up K

a. Estimation: Estimate \(7\frac{4}{5} \div 3\frac{3}{4}\) by rounding each mixed number to the nearest whole number and then dividing.

b. Estimation: Choose the more reasonable estimate for the temperature on a cold winter day: 31°F or 31°C.

c. Measurement: How many meters are in one kilometer? … in one tenth of a kilometer?

d. Percent: What is 50% of $10? … 25% of $10? … 10% of $10?

e. Percent: The calculator is on sale for 25% off the regular price of $10. What is the sale price?

f. Number Sense: Write these numbers in order from least to greatest: 0.02, 0.20, 0.19.

g. Calculation: \(\frac{1}{3}\) of 60, \(\times\) 2, \(+\) 2, \(\div\) 6, \(\times\) 4, \(+\) 2, \(\div\) 2

h. Roman Numerals: Compare: XLIV \(\bigcirc\) 45

problem solving

Choose an appropriate problem-solving strategy to solve this problem.

Baseboard is a material that can be placed where the floor meets a wall. The outer edges of the scale drawing to the right indicate walls. The open spaces in the wall represent doors where baseboard is not used. Use your ruler to determine how many meters of baseboard are needed for the room represented by the scale drawing.
Each place in our decimal number system is assigned a particular value. The value of each place is 10 times greater each time we move one place to the left. So when we multiply a number by 10, the digits all shift one place to the left. For example, when we multiply 34 by 10, the 3 shifts from the tens place to the hundreds place, and the 4 shifts from the ones place to the tens place. We fill the ones place with a zero.

\[
\begin{array}{c}
3 & 4 \\
\downarrow & \downarrow \\
3 & 4 & 0. & (10 \times 34 = 340)
\end{array}
\]

Shifting digits to the left can help us quickly multiply decimal numbers by 10, 100, or 1000. Here we show a decimal number multiplied by 10.

\[
\begin{array}{c}
0 & . & 3 & 4 \\
\downarrow & \downarrow \\
3 & . & 4 & (10 \times 0.34 = 3.4)
\end{array}
\]

We see that the digit 3 moved to the other side of the decimal point when it shifted one place to the left. The decimal point holds steady while the digits move. Although it is the digits that change places when the number is multiplied by 10, we can produce the same result by moving the decimal point in the opposite direction.

\[
\begin{array}{c}
\text{Shift the digits to the left.} & \text{or} & \text{Shift the decimal point to the right.}
\end{array}
\]

\[
\begin{array}{c}
0 & . & 3 & 4 \\
\downarrow & \downarrow \\
3 & . & 4 & (10 \times 0.34 = 3.4)
\end{array}
\]

When we multiply by 10, we may simply shift the decimal point one place to the right.

Since 100 is \(10 \times 10\), multiplying by 100 is like multiplying by 10 \textit{twice}. When we multiply by 100, we may shift the decimal point \textit{two} places to the right.

Since 1000 is \(10 \times 10 \times 10\), we may shift the decimal point \textit{three} places to the right when we multiply by 1000.

\textbf{The number of places we shift the decimal point is the same as the number of zeros we see in 10, 100, or 1000.}
Lesson 111

Example

Multiply: $1.234 \times 100$

To multiply mentally by 100, we may shift the decimal point two places to the right. The product is $123.4$.

$1.234 \times 100 = 123.4$

Generalize Why did we shift the decimal point two places to the right?

Lesson Practice

Multiply:

a. $1.234 \times 10$

b. $1.234 \times 1000$

c. $0.1234 \times 100$

d. $0.345 \times 10$

e. $0.345 \times 100$

f. $0.345 \times 1000$

g. $5.67 \times 10$

h. $5.67 \times 1000$

i. $5.67 \times 100$

Written Practice

Distributed and Integrated

1. In three classrooms there were 23 students, 25 students, and 30 students. If the students in the three classrooms were rearranged so that there were an equal number of students in each room, how many students would there be in each classroom?

2. Composer Duke Ellington was born in 1899. Composer John Williams was born 33 years later. When was John Williams born?

3. a. Write the reduced fraction equal to 25%.

b. Write the reduced fraction equal to 50%.

4. a. List the first six multiples of 6.

b. List the first four multiples of 9.

c. Which two numbers appear in both lists?

5. Connect Name the shaded portion of this square as a percent, as a decimal number, and as a reduced fraction.
6. **Multiple Choice** Which is the shape of a basketball?
   - A cylinder
   - B sphere
   - C cone
   - D circle

7. How many months are in $1\frac{1}{2}$ years?

8. a. How many units long is the perimeter of this shape?
   b. How many square units is the area of this shape?

9. $QR$ is 45 mm. $RS$ is one third of $QR$. $QT$ is 90 mm. Find $ST$.

   ![Diagram of QRSST]

For problems 10 and 11, multiply mentally by shifting the decimal point.

*10. $1.23 \times 10$
*11. $3.42 \times 1000$

*12. **Represent** Use words to name this sum:
   $15 + 9.67 + 3.292 + 5.5$

*13. $4.3 - 1.21$
*14. $0.14 \times 0.6$

*15. $48 \times 0.7$

*16. $0.735 \times 10^2$

17. **Analyze** Write a fraction equal to $\frac{3}{4}$ that has the same denominator as $\frac{3}{8}$. Then add the fraction to $\frac{3}{8}$. Remember to convert your answer to a mixed number.

18. $16 \div 4000$
19. $18.00 \div 10$

20. $\frac{7}{11} + \frac{8}{11}$
21. $\frac{3}{12} + \frac{1}{12}$
22. $5 \frac{9}{10}$

23. $\frac{7}{2} \times \frac{1}{2}$
24. $\frac{2}{3} \div \frac{1}{4}$
25. $3 \div \frac{3}{4}$

26. Compare: $\sqrt{9} + \sqrt{16}$, $\sqrt{9} + 16$
27. The names of two of the 12 months begin with the letter A. What percent of the names of the months begin with the letter A?

28. Elizabeth studied this list of flights between Los Angeles and Philadelphia. Refer to this list to answer parts a and b.

<table>
<thead>
<tr>
<th>Los Angeles to Philadelphia</th>
<th>Philadelphia to Los Angeles</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Depart</strong></td>
<td><strong>Arrive</strong></td>
</tr>
<tr>
<td>6:15 a.m.</td>
<td>2:34 p.m.</td>
</tr>
<tr>
<td>10:10 a.m.</td>
<td>6:33 p.m.</td>
</tr>
<tr>
<td>12:56 p.m.</td>
<td>9:15 p.m.</td>
</tr>
<tr>
<td>3:10 p.m.</td>
<td>11:19 p.m.</td>
</tr>
<tr>
<td><strong>Depart</strong></td>
<td><strong>Arrive</strong></td>
</tr>
<tr>
<td>7:55 a.m.</td>
<td>10:41 a.m.</td>
</tr>
<tr>
<td>10:00 a.m.</td>
<td>12:53 p.m.</td>
</tr>
<tr>
<td>1:30 p.m.</td>
<td>4:17 p.m.</td>
</tr>
<tr>
<td>5:40 p.m.</td>
<td>8:31 p.m.</td>
</tr>
</tbody>
</table>

a. Elizabeth wants to arrive in Philadelphia before 8 p.m. However, she does not want to wake up very early to catch a flight. Which departure time is Elizabeth likely to choose?

b. For her return flight, Elizabeth would like to leave as late as possible and still arrive in Los Angeles by 9:00 p.m. Which departure time is Elizabeth likely to choose?

29. A classroom bookshelf contains 27 books. Eleven of the books are reference books. Five of the books are fiction books. How many books on the bookshelf are not reference or fiction books?

30. At Franklin Elementary School, the first recess of the morning lasts for \( \frac{1}{2} \) of \( \frac{1}{2} \) of an hour. What fraction of an hour is the length of the first recess? How many minutes long is that recess?

To view a slide of an amoeba, Kymma sets a microscope to enlarge objects to 100 times their actual size.

a. If the actual diameter of the amoeba is 0.095 mm, then what is its diameter as seen through the microscope?

b. If Kymma sets the microscope to enlarge objects to 10 times their actual size, what would the diameter of the amoeba appear to be for that setting?

c. If Kymma sets the microscope to enlarge objects to 1000 times their actual size, what would the diameter of the amoeba appear to be in centimeters?
• Finding the Least Common Multiple of Two Numbers

**Power Up**

Power Up K

**facts**

**mental math**

a. **Estimation**: Estimate the cost of 98 tickets that cost $2.50 each.

b. **Measurement**: Elsa was feeling ill. Her fever was 100.7°F. How many degrees was Elsa’s fever above her normal temperature of 98.6°F?

c. **Measurement**: The liquid medicine dropper can hold 1 milliliter of liquid. How many full droppers equal half a liter?

d. **Fractional Parts**: What is \(\frac{1}{10}\) of 30? \(\ldots\) \(\frac{3}{10}\) of 30? \(\ldots\) \(\frac{9}{10}\) of 30?

e. **Probability**: The box contains equal amounts of three flavors of dog treats: peanut butter, vegetable, and chicken. If Grey pulls one dog treat from the box without looking, what is the probability it will *not* be chicken?

f. **Geometry**: If the area of a square is 9 cm\(^2\), what is the length of each side?

g. **Calculation**: \(\sqrt{100}\), \(\div 2\), \(\times 7\), \(+ 1\), \(\div 6\), \(\times 4\), \(\div 2\)

h. **Roman Numerals**: Compare: 96 □ XCIV

**problem solving**

Choose an appropriate problem-solving strategy to solve this problem. Fernando dropped a rubber ball and found that each bounce was half as high as the previous bounce. He dropped the ball from 8 feet, measured the height of each bounce, and recorded the results in a table. Copy this table and complete it through the fifth bounce.

<table>
<thead>
<tr>
<th>Heights of Bounces</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>First</td>
<td>4 ft</td>
</tr>
<tr>
<td>Second</td>
<td></td>
</tr>
<tr>
<td>Third</td>
<td></td>
</tr>
<tr>
<td>Fourth</td>
<td></td>
</tr>
<tr>
<td>Fifth</td>
<td></td>
</tr>
</tbody>
</table>
Here we list the first few multiples of 4 and 6:

Multiples of 4: 4, 8, 12, 16, 20, 24, 28, 32, 36, …

Multiples of 6: 6, 12, 18, 24, 30, 36, …

We have circled the multiples that 4 and 6 have in common. The smallest number that is a multiple of both 4 and 6 is 12.

The smallest number that is a multiple of two or more numbers is called the **least common multiple** of the numbers. The letters $LCM$ are sometimes used to stand for **least common multiple**.

**Example**

Find the least common multiple (LCM) of 6 and 8.

We begin by listing the first few multiples of 6 and 8. Then we circle the multiples they have in common.

Multiples of 6: 6, 12, 18, 24, 30, 36, 42, 48, …

Multiples of 8: 8, 16, 24, 32, 40, 48, …

As we see above, the least of the common multiples of 6 and 8 is 24.

**Prime Numbers on a Hundred Number Chart**

Material needed:

- **Lesson Activity 21**

The first prime number is 2 because 2 has two different factors, but 1 has only one factor. Every even number greater than 2 (such as 4, 6, 8, and so on) is a composite number. Since all even numbers are multiples of 2, they have at least 3 factors—the number itself, the number 1, and 2. On a hundred number chart, we can find the prime numbers and cross out the composite numbers, which are all multiples of prime numbers.
On this hundred number chart, we circled 2 and began crossing out multiples of 2. On Lesson Activity 21, find all the prime numbers. Circle 2 and cross out the multiples of 2. Then circle the next prime number, 3, and cross out the remaining multiples of 3. Then move on to 5, and continue the process until you have found all the prime numbers less than 100.

Lesson Practice

Find the least common multiple (LCM) of each pair of numbers:

a. 2 and 3
   6

b. 3 and 5
   15

c. 5 and 10
   10

d. 2 and 4
   4

e. 3 and 6
   6

f. 3 and 10
   30

g. The denominators of $\frac{5}{8}$ and $\frac{3}{10}$ are 8 and 10. What is the least common multiple of 8 and 10?
   40

h. Use color tiles to make factor arrays for 13 and 15.
   Which number is prime and which number is composite?

Written Practice

1. **Estimate** (77) A small car weighs about one ton. Most large elephants weigh four times that much. About how many pounds would a large elephant weigh?

   about 8000 pounds

2. **Estimate** (74) At one time, the Arctic Ocean was almost completely covered by a polar ice cap, which measured up to 10 feet thick. About how many inches thick was the polar ice cap at that time?

   about 120 inches
3. What is the total cost of 10 movie tickets priced at $5.25 each?

4. Which digit in 375.246 is in the hundredths place?

5. Represent Draw a pentagon. Then draw a reflection of your figure.

6. Write 12.5 as a mixed number.

7. Connect Name the shaded portion of the square at right as a percent, as a decimal number, and as a reduced fraction.

8. Name the shape of an aluminum can.

9. Stefano practiced playing the trombone for 20 minutes on Monday. On Wednesday he practiced for 10 minutes more than he did on Monday. On Friday he practiced for 5 fewer minutes than on Wednesday. How many minutes did Stefano practice on Friday?

10. Find the least common multiple (LCM) of 6 and 9.

11. If $OM$ measures 15 mm, then what is the measure of $LN$?

12. WX is 4.2 cm. XY is 3 cm. WZ is 9.2 cm. Find YZ.

13. $4.38 + 7.525 + 23.7 + 9$

14. $5 - (4.3 - 0.21)$

15. $3.6 \times 40$

16. $0.15 \times 0.5$

17. $10 \times 0.125$

18. $4w = 300$

19. $40 \sqrt{3000}$
20. \( (94) \frac{20}{25} \div 3300 \)

21. \( (63, 75) \frac{33}{7} + \left( 5 - \frac{12}{7} \right) \)

22. \( (41, 86) \frac{1}{2} - \left( 3 \times \frac{1}{2} \right) \)

23. Write fractions equal to \( \frac{1}{4} \) and \( \frac{2}{3} \) that have denominators of 12. Then subtract the smaller fraction from the larger fraction.

**24.** Use this grid to answer parts a and b.

![Grid](image)

a. Name the coordinates of the vertices of triangle \( ABC \).

b. If triangle \( ABC \) were rotated 90º clockwise around point \( C \), then what would be the coordinates of vertex \( A \)?

25. \( (78) \) Compare: \( 3^2 + 4^2 \bigcirc 5^2 \)

**26.** Find the percent equivalent of \( \frac{1}{8} \) by multiplying 100% by \( \frac{1}{8} \). Write the result as a mixed number with the fraction reduced.

27. The lowest temperature ever recorded in North Dakota was \(-60°F\). In Montana the lowest temperature ever recorded was \(-70°F\). Is a temperature of \(-60°F\) warmer or colder than a temperature of \(-70°F\)? How many degrees warmer or colder?
Karen’s flight schedule between Oklahoma City and Indianapolis is shown below. Refer to this schedule to answer parts a–c.

<table>
<thead>
<tr>
<th>Flight</th>
<th>Time</th>
<th>Departure</th>
<th>Arrival</th>
</tr>
</thead>
<tbody>
<tr>
<td>FLIGHT 41</td>
<td>Thu, Aug 22</td>
<td>6:11 a.m. to 8:09 a.m. plane change</td>
<td>Oklahoma City (OKC)</td>
</tr>
<tr>
<td>FLIGHT 11</td>
<td>Thu, Aug 22</td>
<td>9:43 a.m. to 10:38 a.m.</td>
<td>Chicago (ORD)</td>
</tr>
<tr>
<td>FLIGHT 327</td>
<td>Thu, Aug 29</td>
<td>9:58 a.m. to 11:03 a.m. plane change</td>
<td>Indianapolis (IND)</td>
</tr>
<tr>
<td>FLIGHT 337</td>
<td>Thu, Aug 29</td>
<td>12:04 p.m. to 1:33 p.m.</td>
<td>St Louis (STL)</td>
</tr>
</tbody>
</table>

a. Flight 41 of Karen’s trip to Indianapolis takes her to Chicago. How much time is in the schedule for changing planes in Chicago?

b. The times listed in the schedule are gate-to-gate times, from the time the plane pushes away from the gate at departure to the time the plane pulls into the gate at arrival. Find the total of the gate-to-gate times for the two flights from Oklahoma City to Indianapolis.

c. **Explain** The total sum of the gate-to-gate times for the two return flights to Oklahoma City is how many minutes less than for the outbound flights? What might account for the difference in travel time?

29. In Ms. Adrian’s math class, the students spent \( \frac{1}{12} \) of an hour correcting homework and \( \frac{5}{12} \) of an hour working at the board. In simplest form, what fraction of an hour did the students spend doing those tasks?

30. Lionel chopped \( \frac{3}{4} \) of a cup of celery, but he needed to use only \( \frac{1}{2} \) of that amount in a cream soup recipe. What amount of chopped celery did the recipe require?
• Writing Mixed Numbers as Improper Fractions

Power Up

facts

Power Up K

mental math

a. **Measurement:** The swimming pool holds a maximum of 12,000 gallons of water. Carole has already put about 5500 gallons into the pool. About how many more gallons of water are needed to fill the pool?

b. **Number Sense:** Simplify the fractions $\frac{8}{12}, \frac{9}{12}, \text{ and } \frac{15}{12}$.

c. **Percent:** 25% of 12

d. **Percent:** 50% of 19

e. **Percent:** 75% of 12

f. **Geometry:** A hectare is an area of land equivalent to a square that is 100 meters on each side. How many hectares is a field that is 200 meters on each side?

g. **Calculation:** $\frac{1}{6}$ of 24, $\times 5$, $+ 1$, $\div 3$, $\times 8$, $- 2$, $\div 9$

h. **Roman Numerals:** Compare: MD $\bigcirc$ 2000

problem solving

Choose an appropriate problem-solving strategy to solve this problem. Blake is saving money for a new telescope. In January Blake saved $10. In the months February through May, he saved $35 each month. By the end of August, Blake will have doubled the total amount of money he had at the end of May. At that time, will Blake have enough money to purchase a telescope that costs $280? Explain your reasoning.
The picture below shows $1\frac{1}{2}$ shaded circles. How many half circles are shaded?

Three halves are shaded. We may name the number of shaded circles as the mixed number $1\frac{1}{2}$ or as the improper fraction $\frac{3}{2}$.

$$1\frac{1}{2} = \frac{3}{2}$$

We have converted improper fractions to mixed numbers by dividing. In this lesson we will practice writing mixed numbers as improper fractions. We will use this skill later when we learn to multiply and divide mixed numbers.

To help us understand changing mixed numbers into fractions, we can draw pictures. Here we show the number $2\frac{1}{4}$ using shaded circles:

To show $2\frac{1}{4}$ as an improper fraction, we divide the whole circles into the same-size pieces as the divided circle. In this example we divide each whole circle into fourths.

Now we count the total number of fourths that are shaded. We see that $2\frac{1}{4}$ equals the improper fraction $\frac{9}{4}$.

**Example 1**

Name the number of shaded circles as an improper fraction and as a mixed number.

To show the improper fraction, we divide the whole circles into the same-size pieces as the divided circle (in this case, halves). The improper fraction is $\frac{5}{2}$. The mixed number is $2\frac{1}{2}$.
Example 2

Change $2\frac{1}{3}$ to an improper fraction.

One way to find an improper fraction equal to $2\frac{1}{3}$ is to draw a picture that illustrates $2\frac{1}{3}$.

We have shaded 2 whole circles and $\frac{1}{3}$ of a circle. Now we divide each whole circle into thirds and count the total number of thirds.

\[
\frac{3}{3} + \frac{3}{3} + \frac{1}{3} = \frac{7}{3}
\]

We see that seven thirds are shaded, so an improper fraction equal to $2\frac{1}{3}$ is $\frac{7}{3}$.

It is not necessary to draw a picture. We could remember that each whole is $\frac{3}{3}$. So the 2 of $2\frac{1}{3}$ is equal to $\frac{3}{3} + \frac{3}{3}$, which is $\frac{6}{3}$. Then we add $\frac{6}{3}$ to $\frac{1}{3}$ and get $\frac{7}{3}$.

Lesson Practice

For problems a–c, name the number of shaded circles as an improper fraction and as a mixed number:

a. 

b. 

c. 

Change each mixed number to an improper fraction:

d. $4\frac{1}{2}$
e. $1\frac{2}{3}$
f. $2\frac{3}{4}$
g. $3\frac{1}{8}$
1. On a five-day trip, the Jansens drove 1400 miles. What was the average number of miles the Jansens drove on each of the five days?

2. Estimate Round both 634 and 186 to the nearest hundred to estimate their product before multiplying.

3. a. \[ \frac{1}{10} = \square \]

   b. What percent equals the fraction \( \frac{1}{10} \)?

4. The weight of an object on the moon is about \( \frac{1}{6} \) of the weight of the same object on Earth. A person who weighs 108 pounds on Earth would weigh about how many pounds on the moon?

*5. Connect Name the total number of shaded circles as an improper fraction and as a mixed number.

6. An inch is about 2.5 centimeters.

   a. What is the perimeter of this square in inches? In centimeters?

   b. What is the area of this square in square inches? In square centimeters?

*7. What fraction of a year is 3 months? What percent of a year is 3 months?

8. a. Name the shape at right.

   b. How many faces does the shape have?

*9. The denominators of \( \frac{1}{6} \) and \( \frac{1}{4} \) are 6 and 4. What is the least common multiple (LCM) of the denominators?
10. **Connect** To what mixed number is the arrow pointing?

![Diagram showing a number line with arrows pointing to 5 and 6.]

11. \(4.239 + 25 + 6.79 + 12.5\)

12. \(6.875 - (4 - 3.75)\)

13. \(3.7 \times 0.8\)

14. \(0.125 \times 100\)

15. \(0.32 \times 0.04\)

16. \(\frac{408}{17}\)

17. \(27 \div 705\)

18. \(5 \div 17.70\)

19. \(3 \frac{7}{10} + 4\)

20. \(\frac{5}{8} + \frac{1}{8}\)

21. \(-4 \frac{3}{10}\)

22. \(\frac{5}{6} \times 4\)

23. \(\frac{3}{8} \times \frac{1}{2}\)

24. \(\frac{3}{8} \div \frac{1}{2}\)

25. Josette spent \(\frac{1}{6}\) of an hour walking to school and \(\frac{1}{4}\) of an hour walking home from school. How many minutes did Josette spend walking to and from school? What fraction of an hour did Josette spend walking to and from school? (Hint: Write fractions equal to \(\frac{1}{6}\) and \(\frac{1}{4}\) that have denominators of 12. Then add the fractions.)

26. a. What is the volume of a chest of drawers with the dimensions shown?

b. What is the area of the top of the chest?

c. What is the perimeter of the top of the chest?

27. **Explain** Tiana mailed two packages at the post office. One package weighed \(2 \frac{1}{3}\) pounds, and the other weighed \(3 \frac{3}{4}\) pounds. The clerk told Tiana that the total weight of the packages was exactly 6 pounds. Was the clerk correct? Explain your answer.
Lillian is planning a trip from San Diego to San Luis Obispo. The schedules for the trains she plans to take are printed below. Use this information to answer parts a–c.

<table>
<thead>
<tr>
<th>Station</th>
<th>#29</th>
<th>#48</th>
</tr>
</thead>
<tbody>
<tr>
<td>San Diego</td>
<td>Dp 9:30</td>
<td>Ar 7:50</td>
</tr>
<tr>
<td>Anaheim</td>
<td>Ar 11:26</td>
<td>5:51 p.m.</td>
</tr>
<tr>
<td>Los Angeles</td>
<td>12:30 p.m.</td>
<td>4:55 p.m.</td>
</tr>
<tr>
<td>Ventura</td>
<td>2:21 p.m.</td>
<td>2:39 p.m.</td>
</tr>
<tr>
<td>Santa Barbara</td>
<td>3:10 p.m.</td>
<td>1:40 p.m.</td>
</tr>
<tr>
<td>Solvang</td>
<td>4:05 p.m.</td>
<td>12:45 p.m.</td>
</tr>
<tr>
<td>San Luis Obispo</td>
<td>5:30 p.m.</td>
<td>Ar 11:10 a.m.</td>
</tr>
<tr>
<td>Paso Robles</td>
<td>Ar 6:20</td>
<td>Dp 10:00 a.m.</td>
</tr>
</tbody>
</table>

a. The trip from San Diego to San Luis Obispo takes how long?

b. Train #48 departs Santa Barbara 15 minutes after it arrives. At what time does the train depart from Santa Barbara?

c. **Multiple Choice** The distance between San Diego and San Luis Obispo is about 320 miles. From departure to arrival, the train travels about how many miles each hour?
   - A 30 miles
   - B 40 miles
   - C 50 miles
   - D 60 miles

The girls' softball team held a fundraiser by selling calendars. Reyna sold twice as many calendars as Mackenzie, and Cherise sold four more calendars than Reyna. Mackenzie sold ten calendars. How many calendars did Cherise sell?

Use the table to solve parts a and b.

<table>
<thead>
<tr>
<th>Number of School Days per Year (by country)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Country</td>
</tr>
<tr>
<td>---------</td>
</tr>
<tr>
<td>China</td>
</tr>
<tr>
<td>Japan</td>
</tr>
<tr>
<td>Korea</td>
</tr>
<tr>
<td>United States</td>
</tr>
</tbody>
</table>

a. Find the median of the data.

b. Find the range of the data.
• Using Formulas

Power Up

facts
Power Up K

mental math
a. Number Sense: Matthew spent $2\frac{1}{2}$ hours doing homework on Monday, $1\frac{1}{2}$ hours on Tuesday, and 2 hours on Wednesday. What was the average amount of time he spent per day on homework?

b. Measurement: It took the turtle one minute to travel $2\frac{3}{4}$ feet. How many inches is $2\frac{3}{4}$ feet?

c. Fractional Parts: $\frac{1}{8}$ of 24

d. Fractional Parts: $\frac{3}{8}$ of 24

e. Fractional Parts: $\frac{5}{8}$ of 24

f. Powers/Roots: $4^3$

g. Calculation: 25% of 40, $+ 2$, $\times 2$, $+ 1$, $\div 5$, $\times 3$, $+ 1$, $\div 8$, $- 2$

h. Roman Numerals: Compare: MDXX $\bigcirc$ 1520

problem solving
Choose an appropriate problem-solving strategy to solve this problem. Jamisha had 24 square tiles on her desk. She arranged them into a rectangle made up of one row of 24 tiles. Then she arranged them into a new rectangle made up of two rows of 12 tiles.

Draw two more rectangles Jamisha could make using all 24 tiles.

New Concept
Formulas describe processes for solving certain types of problems. Formulas often use letters and other symbols to show the relationship between various measures.
In the examples that follow, we use formulas to solve problems about perimeter, area, and volume.

**Example 1**

The Jacksons have added a dining room to a corner of their house. Mr. Jackson purchased crown molding that will be installed at the intersection of the walls and the ceiling of his dining room. Crown molding costs $5 per foot to install. What will be Mr. Jackson's cost for having the crown molding installed?

Crown molding is installed around the perimeter of the room. We can use the perimeter formula to determine the total length of crown molding and then multiply that length by $5 to find the cost of installation.

\[ P = 2l + 2w \]
\[ P = 2(15\text{ ft}) + 2(12\text{ ft}) \]
\[ P = 54\text{ ft} \]

The perimeter is 54 ft, so the cost of the installed molding is $5 \times 54\text{ ft}$, which is $270.$

**Verify** Why is the perimeter recorded in feet and not in square feet?

**Example 2**

Mrs. Jackson wants to buy carpet for the dining room floor. How many square feet of carpet are needed to cover the floor?
The carpet covers the floor area of the room, so we use the area formula to determine the amount of carpet needed.
\[ A = l \times w \]
\[ A = 15 \text{ ft} \times 12 \text{ ft} \]
\[ A = 180 \text{ sq. ft} \]
Mrs. Jackson will need **180 sq. ft** of carpet to cover the floor.

**Analyze**  The carpet Mrs. Jackson chose costs $5 per square foot. How much will the carpet cost?

**Example 3**

To heat and cool the new room, the Jacksons need to know the volume of the room. How many additional cubic feet of air need to be heated or cooled?

We use the volume formula to determine the amount of cubic feet added to the house.
\[ V = l \times w \times h \]
\[ V = 15 \text{ ft} \times 12 \text{ ft} \times 10 \text{ ft} \]
\[ V = 1800 \text{ cu. ft} \]
The Jacksons have added **1800 cu. ft** of air to heat or cool.

**Verify**  Why is the answer recorded in cubic feet and not in square feet?
Example 4

The Jacksons’ son, Demont, has a trunk in his room for storing toys.

![Trunk Diagram]

a. Mrs. Jackson plans to put a liner on the floor of the trunk. Choose a formula and use it to decide how much area the liner will cover.

b. Mrs. Jackson also plans to paste a border around the entire trunk. Choose a formula and use it to determine the minimum length of border she needs to buy.

a. The shape of the floor of the trunk is a rectangle. We use the area formula to find the area of the 36 in. by 30 in. rectangle.

\[ A = l \times w \]
\[ A = 36 \text{ in.} \times 30 \text{ in.} \]
\[ A = 1080 \text{ sq. in.} \]

b. The border is pasted to the perimeter of the trunk, so we find the perimeter of a 36 in. by 30 in. rectangle.

\[ P = 2l + 2w \]
\[ P = 2(36 \text{ in.}) + 2(30 \text{ in.}) \]
\[ P = 132 \text{ in.} \]

Mrs. Jackson needs at least 132 in. of border.

Lesson Practice

Refer to the diagrams in the examples of this lesson to help you answer problems a and b. For each practice problem, show the formula you can use.

a. The Jacksons want to cover one 12-foot long wall of the dining room with wallpaper. How many square feet will the wallpaper need to cover?

b. Calculate the storage capacity of Demont’s toy box in cubic feet. (Hint: 30 inches is 2.5 feet.)
c. The diagram below is a top view of the Jackson house showing the outside walls. The dashes show the outside walls of the new dining room. Calculate the perimeter of the house.

\[ P = 2l + 2w \]

Written Practice

1. **Represent** (Inv. 3, 37) Draw a circle and shade all but \( \frac{1}{3} \) of it. What percent of the circle is shaded?

2. **Multiple Choice** (74) Which of these units of length would probably be used to measure the length of a room?
   - A inches
   - B feet
   - C miles
   - D light-years

*3. **Multiple Choice** (105) Which of these does not show a line of symmetry?
   - A
   - B
   - C
   - D

*4. **Explain** (21) Garcia’s car can travel 28 miles on one gallon of gas. How far can his car travel on 16 gallons of gas? Explain why your answer is reasonable.

5. Write \( \frac{3}{4} \) as an improper fraction.

6. **Explain** (79, 81) Is it possible for one friend to eat \( \frac{1}{3} \) of a sandwich and for another friend to eat \( \frac{5}{6} \) of the same sandwich? Explain why or why not.
7. The denominators of \( \frac{3}{8} \) and \( \frac{5}{6} \) are 8 and 6. What is the least common multiple (LCM) of the denominators?

8. Refer to this spinner to answer parts a and b.

a. What fraction names the probability that with one spin the spinner will stop on sector A?

b. What is the probability that with one spin the spinner will stop on sector B?

9. QS is 6 cm. RS is 2 cm. RT is 6 cm. Find QT.

10. \( 45 + 16.7 + 8.29 + 4.325 \)

11. \( 4.2 - (3.2 - 1) \)

12. \( 0.75 \times 0.05 \)

13. \( 0.6 \times 38 \)

14. \( 100 \times 7.5 \)

15. \( \$24.36 \div 12 \)

16. \( 4600 \div 5^2 \)

17. \( \frac{9}{10} - \frac{1}{10} \)

18. \( \frac{4}{8} \)

19. \( 4 \times \frac{1}{8} \)

20. At a practice baseball game there were 18 players and 30 spectators. What was the ratio of players to spectators at the game?

21. Connect What percent of the rectangle is shaded?

What percent of the rectangle is not shaded?

22. Write the reduced fraction equal to 60%.

23. Write the reduced fraction equal to 70%.
24. a. **Analyze** A loop of string can be arranged to form a rectangle that is 12 inches long and 6 inches wide. If the same loop of string is arranged to form a square, what would be the length of each side of the square?

b. What is the area of the rectangle pictured in part a?

c. What is the area of the square described in part a?

*25. Find the percent equivalent to $\frac{1}{6}$ by multiplying 100% by $\frac{1}{6}$ and writing the answer as a mixed number with the fraction reduced.

26. **Explain** What is the result of doubling $7\frac{1}{2}$ and dividing the product by 3? Explain why your answer is reasonable.

27. In Duluth, Minnesota, the average January high temperature is 18°F. The average January low temperature is –1°F. How many degrees greater is a temperature of 18°F than a temperature of –1°F?

*28. Each morning of a school day, Chelsea’s alarm wakes her up at a quarter past six, and she leaves for school at a quarter to eight. What mixed number represents the number of hours Chelsea spends on those mornings getting ready for school?

*29. **Explain** The baseball cleats that Orin purchased online arrived in a shoe box. The box measured $11\frac{3}{8}$ in. by $8\frac{3}{4}$ in. by 4 in. Estimate the volume of the box, and then explain why your estimate is reasonable.

*30. Two squares form this hexagon. Refer to this figure for parts a and b.

   a. What is the area of each square?

   b. Combine the areas of the two squares to find the area of the hexagon

---

**Early Finishers**

Real-World Connection

A community center is planning to build a tennis court. A regulation tennis court has a length of 78 ft and a width of 36 ft. In addition, a space of 12 ft is needed on each side of the court and a clearance of 21 ft is needed on each end of the court. Find the area of the entire ground space needed for the tennis court. Be sure to show your work.
• Area, Part 2

Power Up

facts

Power Up K

mental math

a. **Geometry:** The sides of a square are 5 inches long. What is the perimeter of the square? What is the area of the square?

b. **Geometry:** Two angles of the triangle each measure 58°. The other angle measures 64°. What is the sum of the three angles?

c. **Number Sense:** Linda read 21 pages on Friday, 38 pages on Saturday, and 40 pages on Sunday. What was the average number of pages Linda read per day?

d. **Percent:** 25% of 80

e. **Percent:** 50% of 80

f. **Percent:** 75% of 80

g. **Estimation:** Suzie measured the length of the violin as $23\frac{1}{4}$ inches. Express this length as a mixed measure containing feet and inches.

h. **Roman Numerals:** Compare: 92 ○ LXXXII

problem solving

Choose an appropriate problem-solving strategy to solve this problem. Heather just found out that she won the poetry contest, and she is eager to spread the news among her friends and family. Heather told three people about her accomplishment. Then those three people each told two more people. Then each of those people told two more people. How many people other than Heather have received the news?
Recall that we calculate the area of a rectangle by multiplying its length and width. In this lesson we will calculate the area of figures that can be divided into rectangles.

**Example**

Two rectangles are joined to form a hexagon. What is the area of the hexagon?

The hexagon can be divided into two rectangles. We find the area of each rectangle and then we add the areas to find the area of the hexagon.

\[
\begin{align*}
\text{Area I} & = 7 \text{ ft} \times 5 \text{ ft} = 35 \text{ sq. ft} \\
\text{Area II} & = 4 \text{ ft} \times 3 \text{ ft} = 12 \text{ sq. ft} \\
\text{Combined area} & = 47 \text{ sq. ft}
\end{align*}
\]

**Lesson Practice**

**Represent** Copy each figure on your paper. Then find the area of each figure by dividing it into two rectangles and adding the areas of the parts.

- **a.**
  - 7 m
  - 3 m
  - 4 m

- **b.**
  - 4 in.
  - 5 in.

- **c.**
  - 2 cm
  - 4 cm
  - 6 cm

- **d.**
  - 6 ft
  - 1 ft

**Written Practice**

1. One length of string is 48 inches long. Another length of string is 24 feet long. Find the difference of those lengths.
2. **Connect**  (113) Name the total number of shaded circles below as an improper fraction and as a mixed number.

3. a. What fraction names the probability that with one spin, the spinner will stop on sector A?

   b. What is the probability that with one spin, the spinner will stop on sector B?

4. **Connect**  (38) To what mixed number is the arrow pointing?

5. Lawrenia’s first class of the afternoon begins $1\frac{1}{2}$ hours after 11:40 a.m. What time of the day does her first class of the afternoon begin?

6. **Multiple Choice**  (Inv. 2) Which pair of fractions has the same denominator?

   A $\frac{1}{3} \quad \frac{1}{4}$  
   B $\frac{4}{3} \quad \frac{2}{2}$  
   C $\frac{1}{4} \quad \frac{3}{4}$  
   D $\frac{5}{2} \quad \frac{5}{8}$

7. The denominators of $\frac{2}{5}$ and $\frac{2}{3}$ are 5 and 3. Find the least common multiple (LCM) of the denominators.

8. a. Estimate the perimeter of this rectangle.

   b. Estimate the area of this rectangle.

9. $42.98 + 50 + 23.5 + 0.025$

10. **Represent**  (68, 102) How much greater than 5.18 is 6? Use words to write your answer.

11. $0.375 \times 10$

12. $0.14 \times 0.06$

13. $7.8 \times 19$
14. \(2340 \div 30\)  
15. \(18)\overline{2340}\)  
16. \(7)\overline{8765}\)

17. \(\frac{5}{6} + 1\frac{5}{6}\)  
18. \(\frac{7}{8} - 7\frac{1}{8}\)  
*19. \(\frac{4}{5} \times \frac{2}{3}\)

*20. \(\frac{4}{5} \div \frac{2}{3}\)  
21. \(\frac{2}{5} = \frac{\Box}{15}\)  
22. \(\frac{2}{3} = \frac{\Box}{15}\)

23. In problems 21 and 22 you made fractions equal to \(\frac{2}{5}\) and \(\frac{2}{3}\) with denominators of 15. Add the fractions you made. Remember to convert the answer to a mixed number.

24. \(a.\) What is the perimeter of this regular pentagon?  
\(b.\) Explain how you found your answer for part \(a.\)  
\(c.\) A regular pentagon has how many lines of symmetry?

25. \(a.\) What is the area of this hexagon?  
\(b.\) What is the perimeter of the hexagon?

26. What fraction of a square mile is a field that is \(\frac{1}{2}\) mile long and \(\frac{1}{4}\) mile wide?
27. Sandie says that multiplying $4\frac{1}{2}$ by 3 and then subtracting 1 gives an answer of $11\frac{3}{8}$. Explain how rounding can be used to decide if Sandie’s answer is reasonable.

28. Interpret The table shows the average monthly temperatures during autumn in Caribou, Maine. Display the data in a line graph.

<table>
<thead>
<tr>
<th>Month</th>
<th>Temperature ($^\circ$F)</th>
</tr>
</thead>
<tbody>
<tr>
<td>September</td>
<td>54</td>
</tr>
<tr>
<td>October</td>
<td>43</td>
</tr>
<tr>
<td>November</td>
<td>31</td>
</tr>
</tbody>
</table>

29. a. Does this prism have parallel lines?  
     b. Does this prism have perpendicular lines?

30. Two squares form this hexagon. If the squares were separated, their perimeters would be 12 cm and 24 cm, respectively. However, the perimeter of the hexagon is not the sum of the perimeters of the squares because all of the sides of the small and large squares are not part of the perimeter of the hexagon. Copy the hexagon on your paper, and show the length of each of the six sides. What is the perimeter of the hexagon?

Mr. Rio plans to cover his backyard with grass. The diagram below shows the dimensions of his backyard. How many square yards of grass are needed?
LESSON 116

• Finding Common Denominators to Add, Subtract, and Compare Fractions

Power Up

facts

Power Up K

mental math

a. Geometry: A rectangle is 6 inches long and 4 inches wide. What is its perimeter? What is its area?
b. Time: How many seconds is two and a half minutes?
c. Percent: What is 10% of $300? … 10% of $30? … 10% of $3?
d. Number Sense: $2 − \frac{3}{5}$
e. Fractional Parts: $\frac{1}{2}$ of 81
f. Probability: Jill’s schoolbag contains 2 red pens, 4 black pens, 1 blue pen, and 1 green pen. If Jill selects one pen without looking, what is the probability it will be a black pen? Express this number as a percent.
g. Calculation: $\sqrt{16}, \times 5, − 6, ÷ 7, + 8, \times 9, ÷ 10$
h. Roman Numerals: Compare: CCCIV  340

problem solving

Choose an appropriate problem-solving strategy to solve this problem. The local newspaper sells advertising for $20 per column inch per day. An ad that is 2 columns wide and 4 inches long is 8 column inches ($2 \times 4 = 8$) and costs $160 each day ($8 \times $20). What would be the total cost of running a 3-column by 8-inch ad for two days?
New Concept

The fractions $\frac{1}{4}$ and $\frac{3}{4}$ have common denominators. The fractions $\frac{1}{2}$ and $\frac{1}{4}$ do not have common denominators. Fractions have common denominators if their denominators are equal.

<table>
<thead>
<tr>
<th>Common denominators</th>
<th>Different denominators</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{1}{4} \longleftrightarrow \frac{3}{4}$</td>
<td>$\frac{1}{2} \longleftrightarrow \frac{1}{4}$</td>
</tr>
</tbody>
</table>

To compare, add, or subtract fractions that have different denominators, we first change the name of one or more of the fractions so that they have common denominators. The least common multiple (LCM) of the denominators is the least common denominator of the fractions. The denominators of $\frac{1}{2}$ and $\frac{1}{4}$ are 2 and 4. The LCM of 2 and 4 is 4, so the least common denominator for halves and fourths is 4.

Example 1

In one of Katie’s cookbooks, a recipe for salsa calls for $\frac{3}{4}$ cup of chopped fresh cilantro. A salsa recipe given to Katie by a friend calls for $\frac{7}{8}$ cup of chopped fresh cilantro. Which recipe calls for more cilantro?

Rewriting fractions with common denominators can help us compare fractions. The denominators are 4 and 8. We change fourths to eighths by multiplying by $\frac{2}{2}$.

$$\frac{3}{4} \times \frac{2}{2} = \frac{6}{8}$$

We see that $\frac{6}{8}$ is less than $\frac{7}{8}$. We can also express the comparison with a less than sign:

$$\frac{3}{4} < \frac{7}{8}$$

We find that the recipe from Katie’s friend calls for more cilantro than the cookbook recipe.

Example 2

Add: $\frac{1}{2} + \frac{1}{4}$

Since $\frac{1}{2}$ and $\frac{1}{4}$ have different denominators, we change the name of $\frac{1}{2}$ so that both fractions have a denominator of 4. We change $\frac{1}{2}$ to fourths by multiplying by $\frac{2}{2}$, which gives us $\frac{2}{4}$.
Example 3

Subtract: \[3 \frac{1}{2} - 1 \frac{1}{6}\]

We work with the fraction part of each mixed number first. The denominators are 2 and 6. We can change halves to sixths. We multiply \(\frac{1}{2}\) by \(\frac{3}{3}\) and get \(\frac{3}{6}\).

\[
\frac{1}{2} \times \frac{3}{3} = \frac{3}{6}
\]

Then we subtract and reduce the answer.

\[
\frac{3}{6} - \frac{1}{6} = \frac{2}{6} = 2 \frac{1}{3}
\]

Example 4

Add: \(\frac{1}{3} + \frac{1}{2}\)

For this problem we need to rename both fractions. The denominators are 3 and 2. The LCM of 3 and 2 is 6, so the least common denominator for thirds and halves is sixths. We rename the fractions and then add.

\[
\frac{1}{3} \times \frac{2}{2} = \frac{2}{6}
\]

\[
+ \frac{1}{2} \times \frac{3}{3} = \frac{3}{6}
\]

\[
\frac{5}{6}
\]
Lesson Practice

For problems a–c, find a common denominator and compare.

a. Bart spent \( \frac{7}{12} \) of an hour on math and \( \frac{2}{3} \) of an hour reading. Compare \( \frac{7}{12} \) and \( \frac{2}{3} \) to find whether Bart spent more time on math or reading.

b. Copy these fractions and replace the circle with the correct comparison symbol. \( \frac{2}{5} \bigcirc \frac{1}{3} \)

c. The twins took turns carrying the tent up the mountain. Larry carried the tent \( \frac{5}{10} \) of the distance, and Barry carried it \( \frac{2}{4} \) of the distance. Who carried the tent farther?

For problems d–q, find each sum or difference. As you work the problems, follow these steps:

- Find the common denominator.
- Rename one or both fractions.
- Add or subtract the fractions.
- Reduce the answer when possible.

d. \( \frac{1}{2} + \frac{1}{8} \)  
e. \( \frac{1}{2} - \frac{1}{4} \)  
f. \( \frac{3}{4} + \frac{1}{8} \)

g. \( \frac{2}{3} - \frac{1}{9} \)  
h. \( \frac{1}{3} + \frac{1}{4} \)  
i. \( \frac{1}{2} - \frac{1}{3} \)

j. \( 3\frac{1}{4} \)  
k. \( 2\frac{1}{8} \)  
l. \( 3\frac{1}{2} \)  
m. \( 2\frac{3}{4} \)

\[ + \frac{2}{2} \quad + \frac{5}{2} \quad - \frac{1}{6} \quad - \frac{1}{2} \]

n. \( 5\frac{5}{8} \)  
o. \( 3\frac{1}{2} \)  
p. \( 4\frac{3}{4} \)  
q. \( 4\frac{1}{2} \)

\[ + \frac{1}{4} \quad + \frac{1}{3} \quad - \frac{1}{3} \quad - \frac{1}{5} \]

Written Practice

Distributed and Integrated

\*1. **Represent** [37, 107] Draw a circle. Shade all but \( \frac{1}{6} \) of it. What percent of the circle is shaded?

2. In 1875 Bret Harte wrote a story about the California Gold Rush of 1849. How many years after the Gold Rush did he write the story?
3. a. What is the chance of the spinner stopping on 4 with one spin?
   b. What is the probability that with one spin the spinner will stop on a number less than 4?

4. **Multiple Choice** Which of these does *not* show a line of symmetry?

   ![Diagram](image)

5. Compare these fractions. First write the fractions with common denominators.

   \[
   \frac{2}{3} \bigcirc \frac{5}{6}
   \]

6. **Connect** Name the total number of shaded circles as an improper fraction and as a mixed number.

7. Alberto counted 100 cars and 60 trucks driving by the school. What was the ratio of trucks to cars that Alberto counted driving by the school?

8. a. What is the perimeter of this square?
   b. What is the area of this square?

9. \(AC\) is 70 mm. \(BC\) is 40 mm. \(BD\) is 60 mm. Find the length of \(AD\).

10. \(\frac{1}{4} + \frac{1}{8}\)
11. \(\frac{3}{4} - \frac{1}{2}\)
12. \(\frac{7}{8} - \frac{3}{4}\)
13. \(2\frac{5}{8} - 1\frac{1}{2}\)
14. \(3\frac{1}{2} - 2\frac{1}{8}\)
15. \(5\frac{1}{6} + 1\frac{1}{3}\)
16. \( \frac{3}{5} \times 3 \)

17. \( 3 \div \frac{3}{5} \)

18. \( 6.5 \times 100 \)

19. \( 4.6 \times 80 \)

20. \( 0.18 \times 0.4 \)

21. \( 10 \) \( \frac{13.20}{\text{}} \)

22. \( 12 \) \( \frac{13.20}{\text{}} \)

23. \( 1470 \div 42 \)

24. Which angle in quadrilateral \( ABCD \) is an obtuse angle?

25. Add these fractions. First rename the fractions so that they have a common denominator of 12.

\[ \frac{1}{4} + \frac{2}{3} \]

26. What is the area of this figure?

27. Explain Is it possible to arrange exactly 85 chairs in 12 rows and have the same number of chairs in each row? Explain why or why not.

28. Explain The capacity of the fuel tank on Jim’s car is 12.3 gallons, and Jim can travel an average of 29 miles for every gallon of fuel his car uses. What is a reasonable estimate of the distance Jim can travel with one full tank of fuel? Explain why your estimate is reasonable.

29. Explain These fractions do not add to the same sum.

\[ \frac{2}{3} + \frac{3}{4} \quad \frac{3}{8} + \frac{2}{5} \]

Which sum is greater? Explain how you can compare each addend to \( \frac{1}{2} \) to find the answer.
Tim plans to take the train from Fort Collins, where he attends college, to Union Station in Denver. From Union Station he will take a taxi to a job interview, meet a friend for dinner, and then return to Fort Collins at night. Use this information and the train schedule below to solve parts a–c.

<table>
<thead>
<tr>
<th>Cheyenne • Fort Collins • Denver</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
</tr>
<tr>
<td>11:30 a.m.</td>
</tr>
<tr>
<td>12:30 p.m.</td>
</tr>
<tr>
<td>12:40 p.m.</td>
</tr>
<tr>
<td>1:00 p.m.</td>
</tr>
<tr>
<td>1:35 p.m.</td>
</tr>
<tr>
<td>2:05 p.m.</td>
</tr>
</tbody>
</table>

a. Tim plans to study on the ride south to Denver. How long will he have for studying between the departure from Fort Collins and the arrival in Denver?

b. While he’s in Denver, Tim plans to have dinner with a friend. The drive from the train station to the restaurant takes about 20 minutes. Tim wants to be back at the train station an hour before its departure from Denver. By what time should Tim and his friend leave the restaurant?

c. **Explain** If the college campus is a 5-minute walk from the Fort Collins train station, can Tim be back on campus by midnight? Explain your answer.

Hakib skied three trails last month. The first trail was \(1\frac{3}{4}\) miles long, the second trail was \(3\frac{1}{2}\) miles long, and the third trail was \(4\frac{7}{8}\) miles long.

a. What is the difference between the longest trail and the shortest trail?

b. Altogether, how many miles did Hakib ski last month?
• Dividing a Decimal Number by a Whole Number

**Power Up**

### facts

**Power Up K**

### mental math

a. **Estimation:** Using compatible numbers, estimate the per gallon price for a 29.6 gallon tank that cost $64 to fill with gas.

b. **Time:** The Richardsons left their house at 7:45 a.m. and returned home at 4:15 p.m. How long were they gone?

c. **Time:** The space shuttle makes one orbit of the earth in about 90 minutes. About how long does it take the space shuttle to make 3 orbits?

d. **Percent:** The board game is regularly priced at $20. It is on sale for 10% off. What is the sale price?

e. **Money:** Which coin has a value equal to \( \frac{1}{8} \) of $2?

f. **Measurement:** To make lemonade, Yoshi used 10 cups of water. How many pints of water is 10 cups?

g. **Calculation:** \( \frac{1}{5} \) of 20, \( \times 4, - 4, \div 4, + 4, \times 4 \)

h. **Roman Numerals:** Compare: 110 \( \bigcirc \) XC

### problem solving

Choose an appropriate problem-solving strategy to solve this problem. Find the next three numbers in this sequence:

1, 1, 2, 3, 5, 8, 13, ____, ____, ____ , …
Dividing a decimal number by a whole number is like dividing money by a whole number. The decimal point in the quotient is directly above the decimal point inside the division box.
In the chart below, “÷ by whole (W)” means “division by a whole number.” The memory cue “up” reminds us where to place the decimal in the quotient. (We will later learn a different rule for dividing by a decimal number.)

### Decimals Chart

<table>
<thead>
<tr>
<th>Operation</th>
<th>+ or −</th>
<th>×</th>
<th>÷ by whole (W)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Memory cue</td>
<td>line up</td>
<td>×; then count</td>
<td>up</td>
</tr>
<tr>
<td>± ×</td>
<td>×</td>
<td>W</td>
<td></td>
</tr>
</tbody>
</table>

You may need to...
- Place a decimal point on the end of whole numbers.
- Fill each empty place with a zero.

We sometimes need to use one or more zeros as placeholders when dividing decimal numbers. Here we show this using money.

Suppose $0.12 is shared equally by 3 people. The division is shown below. Notice that the decimal point in the quotient is directly above the decimal point in the dividend. We fill each empty place with a zero and see that each person will receive $0.04.

\[
\begin{array}{c|c|c}
& \$0.40 & \$0.04 \\
\hline
3 & \$0.12 & 3)\$0.12 \\
& 12 & 12 \\
\hline
& 0 & 0 \\
\end{array}
\]

### Example 1

For an art project, Corbin must cut a length of ribbon in half. The ribbon is 4.8 meters long. If he cuts the ribbon correctly, how long will each length of ribbon be?

We are dividing 4.8 meters by 2, which is a whole number. We recall the memory cue “up” and place the decimal point in the answer directly above the decimal point inside the division box. Then we divide.

\[
\begin{array}{c|c|c}
& 2.4 \\
\hline
2 & 4.8 \\
& 4 \\
\hline
& 0.8 \\
\hline
& 0 \\
\end{array}
\]

Each length of ribbon will be **2.4 meters**.
Example 2

Divide: $3 \div 0.42$

We place the decimal point in the answer "straight up." Then we divide.

$3 \overline{)0.42}$

\[ 3 \]

\[ 12 \]

\[ 12 \]

\[ 0 \]

0.14

Example 3

Divide: $0.15 \div 3$

We rewrite the problem using a division box. The decimal point in the answer is "straight up." We divide and remember to fill empty places with zeros.

$3 \overline{)0.15}$

15

0

0.05

Example 4

Divide: $0.0024 \div 3$

We rewrite the problem using a division box. The decimal point in the answer is "straight up." We divide and remember to fill empty places with zeros.

$3 \overline{)0.0024}$

0.0008

Generalize

How is dividing a decimal by a whole number similar to dividing a money amount by a whole number? How is it different?

Lesson Practice

Divide:

a. $4 \div 0.52$

b. $6 \div 3.6$

c. $0.85 \div 5$

d. $5 \div 7.5$

e. $5 \div 0.65$

f. $2.1 \div 3$

g. $4 \div 0.16$

h. $0.35 \div 7$

i. $5 \div 0.0025$

j. $0.08 \div 4$

k. $6 \div 0.24$

l. $0.0144 \div 3$

m. Estimate: A gallon is about 3.78 liters. About how many liters is half a gallon?

Written Practice

*1. Represent: Write the following sentence using digits and symbols:

The sum of one sixth and one third is one half.
2. **Analyze** Gilbert scored half of his team’s points. Socorro scored 8 fewer points than Gilbert. The team scored 36 points. How many points did Socorro score?

3. In the northern hemisphere, the first day of winter is December 21 or 22. The first day of summer is 6 months later. The first day of summer can be on what two dates?

4. **a.** What are all the possible outcomes when the spinner is turned?
   
   **b.** What is the probability that with one spin the spinner will stop on a number greater than one?

   **c.** What is the chance of landing on a three with one spin?

5. **Connect** Name the shaded portion of this rectangle as a fraction, as a decimal, and as a percent.

6. If each side of a regular octagon is 6 inches long, then the perimeter of the octagon is how many feet? What formula could you use?

7. **Represent** Name the total number of shaded circles as an improper fraction and as a mixed number.

8. What is the largest four-digit odd number that uses the digits 7, 8, 9, and 0 once each?

9. Refer to rectangle \(ABCD\) to answer problems a–c. In the rectangle, \(AB\) is 3 cm and \(BC\) is 4 cm.
   
   **a.** Which segment is parallel to \(AB\)?

   **b.** What is the perimeter of the rectangle?

   **c.** What is the area of the rectangle?

10. L’Shawn’s hallway locker measures 12 inches wide by 12 inches deep by 5 feet tall. What is the volume of the locker in cubic feet?
11. $KL$ is 56 mm. $LM$ is half of $KL$. $MN$ is half of $LM$. Find $KN$.

12. $16 + 3.17 + 49 + 1.125$

13. How much greater is 3.42 than 1.242?

14. $4.3 \times 100$

15. $6.4 \times 3.7$

16. $0.36 \times 0.04$

17. $2 \overline{)3.6}$

18. $7 \overline{)0.0049}$

19. $1.35 \times 90$

20. $\frac{21}{8} + 1 \frac{3}{4}$

21. $\frac{1}{3} + 1 \frac{6}{10}$

22. $\frac{7}{10} - \frac{1}{2}$

23. $\frac{3}{10} - \frac{1}{5}$

24. $4 \times \frac{3}{2}$

25. $\frac{3}{4} \div \frac{1}{4}$

26. Reduce: $\frac{18}{144}$

27. Find the sum of $3 \frac{1}{5}$ and $2 \frac{1}{2}$ by first rewriting the fractions with 10 as the common denominator.

28. To finish covering the floor of a room, Abby needed a rectangular piece of floor tile 6 inches long and 3 inches wide.

a. What is the area of this rectangle in square inches?

b. What is the area of the rectangle in square feet?
29. A 2-by-2-inch square is joined with a 5-by-5-inch square to form a hexagon. Refer to the figure to answer parts a and b.

a. What is the area of the hexagon?

b. Copy the hexagon and show the lengths of all six sides. Then find the perimeter of the hexagon.

30. Mahdi is a jewelry designer. She has three irregular 10-karat gold nuggets. The weights of the nuggets are $28\frac{1}{3}$ grams, $56\frac{2}{3}$ grams, and 85 grams. What is the total weight in grams of the nuggets?

Mount Vesuvius is an active volcano located to the east of Naples, Italy. To reach the top of the volcano, visitors must climb to an elevation of 4202.76 feet. A group of hikers start their climbing from sea level (elevation 0) and want to climb Mount Vesuvius in three days.

a. If they want to gain the same amount of elevation each day, how many feet would this be?

b. What if they climb down the mountain in two days? How many feet do they climb down per day?
• More on Dividing Decimal Numbers

Power Up

facts

Power Up K

mental math

a. **Number Sense:** 32 \times 10
b. **Number Sense:** 16 \times 20
c. **Number Sense:** 8 \times 40
d. **Powers/Roots:** 5^3
e. **Measurement:** Two tables are each 48 inches long. If the tables are placed end to end, how many feet long is the resulting table?
f. **Time:** How many years is \( \frac{1}{4} \) of a century?
g. **Number Sense:** 25 − 12\frac{1}{2}
h. **Calculation:** 6^2 − 8, \div 7, \times 2, + 10, \div 2, \div 3

problem solving

Choose an appropriate problem-solving strategy to solve this problem. How many 1-inch cubes would be needed to build a rectangular solid 5 inches long, 4 inches wide, and 3 inches high?

New Concept

We usually do not write remainders with decimal division problems. The procedure we will follow for now is to continue dividing until the “remainder” is zero. In order to continue the division, we may need to attach extra zeros to the decimal number that is being divided. **Remember that attaching extra zeros to the back of a decimal number does not change the value of the number.**
Example 1

Divide: $0.6 \div 5$

The first number goes inside the division box. The decimal point is straight up. As we divide, we attach a zero and continue dividing.

$$
\begin{array}{c}
5 \overline{)0.60} \\
\underline{5}\ \\
10 \\
\underline{10}\ \\
0
\end{array}
$$

Justify Why is 60 hundredths equal to 6 tenths?

Example 2

Divide: $0.3 \div 4$

As we divide, we attach zeros and continue dividing. We fill each empty place in the quotient with a zero.

$$
\begin{array}{c}
4 \overline{)0.300} \\
\underline{28}\ \\
20 \\
\underline{20}\ \\
0
\end{array}
$$

Verify Demonstrate how to check the answer.

Example 3

Divide: $3.4 \div 10$

As we divide, we attach a zero to 3.4 and continue dividing. Notice that the same digits appear in the quotient and dividend, but in different places.

$$
\begin{array}{c}
10 \overline{)3.40} \\
\underline{30}\ \\
40 \\
\underline{40}\ \\
0
\end{array}
$$

Thinking Skill

Generalize How is dividing by 10 similar to multiplying by 10? How is it different?

When we divide a number by 10, we find that the answer has the same digits, but the digits have shifted one place to the right.

$$
\begin{array}{c}
3.4 \div 10 = 0.34 \\
34. \div 10 = 3.4
\end{array}
$$

We can use this pattern to find the answer to a decimal division problem when the divisor is 10. The shortcut is very similar to the method we use when multiplying a decimal number by 10. In both cases it is the digits that are shifting places.
However, we can make the digits appear to shift places by shifting the decimal point instead. To divide by 10, we shift the decimal point one place to the left.

\[3.4 \div 10 = 0.34\]

Dividing by 100 is like dividing by 10 twice. When we divide by 100, we shift the decimal point two places to the left. When we divide by 1000, we shift the decimal point three places to the left. We shift the decimal point the same number of places as there are zeros in the number we are dividing by (10, 100, or 1000). We can remember which way to shift the decimal point if we keep in mind that dividing a number into 10, 100, or 1000 parts produces smaller numbers. As a decimal point moves to the left, the value of the number becomes smaller and smaller.

**Example 4**

Mentally divide 3.5 by 100.

When we divide by 10, 100, or 1000, we can find the answer mentally without performing the division algorithm. To divide by 100, we shift the decimal point two places. We know that the answer will be less than 3.5, so we remember to shift the decimal point to the left. We fill the empty place with a zero.

\[3.5 \div 100 = 0.035\]

**Connect** Explain how to mentally divide 3.5 by 1000.

**Lesson Practice**

Divide:

- a. \[0.6 \div 4\]
- b. \[0.12 \div 5\]
- c. \[0.1 \div 4\]
- d. \[0.1 \div 2\]
- e. \[0.4 \div 5\]
- f. \[1.4 \div 8\]
- g. \[0.5 \div 4\]
- h. \[0.6 \div 8\]
- i. \[0.3 \div 4\]

Mentally perform the following divisions:

- j. \[2.5 \div 10\]
- k. \[32.4 \div 10\]
- l. \[2.5 \div 10\]
- m. \[32.4 \div 100\]
- n. \[2.5 \div 1000\]
- o. \[32.4 \div 1000\]
- p. \[12 \div 10\]
- q. \[12 \div 100\]
- r. \[12 \div 1000\]

**Written Practice**

**Multiple Choice** Which of these shows two parallel line segments that are not horizontal?

A  

B  

C  

D  

*1. Multiple Choice* Which of these shows two parallel line segments that are not horizontal?
2. Byron estimated the product of $6\frac{1}{10}$ and $4\frac{7}{8}$ by first rounding each factor to the nearest whole number. What was his estimate?

3. How many 12¢ pencils can V’Nessa buy with one dollar?

4. **Multiple Choice** Which of these figures does not show a line of symmetry?
   
   
   ![Diagram of figures A, B, C, and D]

   *5. The first roll of the bowling ball knocked down 3 of the 10 pins. What percent of the pins were still standing?*

6. a. Write the fraction equal to 4%.
   
   b. Write the fraction equal to 5%.

7. **Represent** Name the total number of shaded circles as an improper fraction and as a mixed number.

   ![Diagram of shaded circles]

   *8. Rihanna has been asked to divide $1\frac{3}{8}$ cups of wheat flour into two equal amounts. She knows that the improper fraction $\frac{11}{8}$ can be used to represent $1\frac{3}{8}$, and she knows that dividing by 2 is the same as multiplying by $\frac{1}{2}$. How many cups of flour will each of the equal amounts represent?*

9. A stop sign has the shape of an 8-sided polygon. Name a polygon that has 8 sides. Does a stop sign have rotational symmetry?

10. Arrange these numbers in order from least to greatest: $\frac{5}{3}$, $\frac{5}{6}$, $\frac{5}{5}$

11. Neil worked on a task for $1\frac{1}{3}$ hours before taking a break. The task takes $2\frac{3}{4}$ hours to complete. After Neil begins working again, how long will it take him to complete the task?
12. The perimeter of this square is 1.2 meters.
   a. How long is each side of this square?
   b. What is the area of this square?

13. 49.35 + 25 + 3.7

14. Compare: $\sqrt{81} + \sqrt{100} \bigcirc 9^2 + 10^2$

15. Represent Subtract 1.234 from 2. Use words to write the answer.

16. 0.0125 ÷ 5

17. 4.2 × 100

18. 0.5 × 0.17

19. 0.6 ÷ 4

20. 0.6 ÷ 10

21. 4\left(\begin{array}{c}1.8\end{array}\right)

22. $\frac{31}{9} + \frac{1}{3}

23. $\frac{1}{3} + \frac{5}{6}

24. $\frac{7}{8} - \frac{1}{4}

25. $4\frac{1}{2} - 1\frac{3}{10}$

26. $6 \times \frac{2}{3}$

27. $6 \div \frac{2}{3}$

28. Divide mentally:
   a. 3.5 ÷ 100
   b. 87.5 ÷ 10

29. A 2 cm by 3 cm rectangle is joined to a 4 cm by 6 cm rectangle to form this hexagon. Refer to the figure to answer problems a and b.
   a. What is the area of the hexagon?
   b. Copy the hexagon and show the lengths of all six sides. Then find the perimeter of the hexagon.

30. Explain On his way home from work, Carter purchased 3 gallons of milk for $2.19 per gallon and 2 loaves of bread for $1.69 per loaf. What is a reasonable estimate of Carter’s total cost? Explain why your estimate is reasonable.
• Dividing by a Decimal Number

Power Up

Power Up K

a. **Number Sense:** $2 \times 250$

b. **Number Sense:** $4 \times 125$

c. **Estimation:** The textbook is $11\frac{1}{8}$ in. long and $8\frac{1}{8}$ in. wide. Round each length to the nearest inch, and then use your estimates to find the approximate perimeter of the book cover.

d. **Geometry:** What is the area of a patio that is 15 ft long and 10 ft wide?

e. **Percent:** What number is 10% of 20?

f. **Percent:** What number is 10% more than 20?

g. **Percent:** What number is 10% less than 20?

h. **Roman Numerals:** Compare: MCMXCIX ☐ MM

Choose an appropriate problem-solving strategy to solve this problem. Makayla erased some of the digits from a multiplication problem and gave it to Connor as a problem-solving exercise. Copy Makayla’s multiplication problem and find the missing digits for Connor.

New Concept

We have practiced dividing decimal numbers by whole numbers. In this lesson we will practice dividing decimal numbers by decimal numbers.
The two problems below are different in an important way.

\[ \begin{align*}
3) & \quad 0.12 \\
0.3) & \quad 0.12
\end{align*} \]

The problem on the left is division by a whole number. The problem on the right is division by a decimal number.

When dividing by a decimal number with pencil and paper, we take an extra step. Before dividing, we shift the decimal points so that we are dividing by a whole number instead of by a decimal number.

\[ 0.3) \overline{0.12} \]

We move the decimal point of the divisor so that it becomes a whole number. Then we move the decimal point of the dividend the same number of places. The decimal point in the quotient will be straight up from the new location of the dividend’s decimal point. To remember how to divide by a decimal number, we may think, “Over, over, and up.”

\[ \begin{align*}
\text{up} \\
0.3) & \quad 0.12 \\
\text{over} & \quad \text{over}
\end{align*} \]

To help us understand why this procedure works, we will write “0.12 divided by 0.3” with a division bar.

\[ \frac{0.12}{0.3} \]

Notice that we can change the divisor, 0.3, into a whole number by multiplying by 10. So we multiply by \( \frac{10}{10} \) to make an equivalent division problem.

\[ \frac{0.12 \times 10}{0.3 \times 10} = \frac{1.2}{10} \]

Multiplying by \( \frac{10}{10} \) moves both decimal points “over.” Now the divisor is a whole number and we can divide.

\[ 3) \overline{1.2} \]

We will add this memory cue to the decimals chart. In the last column, “÷ by decimal (D)” means “division by a decimal number.”
### Decimals Chart

<table>
<thead>
<tr>
<th>Operation</th>
<th>+ or −</th>
<th>×</th>
<th>÷ by whole (W)</th>
<th>÷ by whole (D)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Memory cue</td>
<td>line up</td>
<td>×; then count</td>
<td>up</td>
<td>over, over, up</td>
</tr>
<tr>
<td>±.</td>
<td>± .</td>
<td>± .</td>
<td>± .</td>
<td>± .</td>
</tr>
</tbody>
</table>

You may need to:
- Place a decimal point on the end of whole numbers.
- Fill each empty place with a zero.

### Example

**Divide: 0.6 \( \div \) 2.34**

We are dividing by the decimal number 0.6. We change 0.6 into a whole number by moving its decimal point “over.” We also move the decimal point in the dividend “over.” The decimal point in the quotient will be “straight up” from the new location of the decimal point in the division box.

```
3.9
---
0.6 | 2.3 4
     |
     |
---
     1 8
     
     5 4
8
5
4
0
```

**Verify** Demonstrate how to check the answer.

### Lesson Practice

**Divide:**

- a. 0.3 \( \div \) 1.2
- b. 0.3 \( \div \) 0.42
- c. 1.2 \( \div \) 0.24
- d. 0.4 \( \div \) 0.24
- e. 0.4 \( \div \) 5.6
- f. 1.2 \( \div \) 3.6
- g. 0.6 \( \div \) 2.4
- h. 0.5 \( \div \) 0.125
- i. 1.2 \( \div \) 2.28

### Written Practice

1. Copy the decimals chart in this lesson.

2. The ages of five neighborhood friends are 9, 8, 7, 6, and 5 years. What is the average age of the friends?

3. At the wildlife park there were lions, tigers, and bears. There were 24 bears. If there were twice as many lions as tigers and twice as many tigers as bears, how many lions were there?
4. Joey has $18.35. Raimi has $22.65. They want to put their money together to buy a car that costs $16,040. How much more money do they need?

$15,999

5. a. **Multiple Choice** Which numbers below do *not* show a line of symmetry?

   A  0  B  3  C  5  D  2

b. Which figure A–D does *not* have rotational symmetry?

*6. Write the mixed number $3\frac{1}{3}$ as an improper fraction. Then multiply the improper fraction by $\frac{3}{4}$. Remember to simplify your answer.

7. **Conclude** Refer to quadrilateral ABCD to answer parts a and b.

   a. Which angle appears to be an obtuse angle?

   b. What type of quadrilateral is quadrilateral ABCD?

*8. 3\frac{1}{2} + 1\frac{1}{3}  
*9. 2\frac{1}{6} + 1\frac{1}{2}  
*10. 5\frac{5}{6} - 1\frac{1}{2}  
*11. 4\frac{2}{3} - 1\frac{1}{4}  

*12. 6\overline{0.0144}  
*13. 5\overline{1.2}  
*14. 12\overline{1800}  

*15. 0.3\overline{0.24}  
*16. 50\overline{1000}  
*17. 1.2\overline{0.180}  

*18. Divide mentally:
   a. 0.5 ÷ 10  
   b. 0.5 ÷ 100

19. (3 − 1.6) − 0.16

20. 0.12 \times 0.30  
*21. 0.12 \times 10  
*22. 75 \times 48

*23. 4 \times \frac{3}{8}  
*24. 4 \div \frac{3}{8}
25. a. What is the perimeter of this rectangle?
   b. What is the area of this rectangle?

26. What is the volume of a room that is 10 feet wide, 12 feet long, and 8 feet high?

27. Two squares are joined to form this hexagon. Refer to the figure to answer parts a and b.
   a. What is the area of the hexagon?
   b. Copy the hexagon and show the lengths of the six sides. Then find the perimeter of the hexagon.

28. Trevor polled fifth graders to find out how many items they put in their backpacks. The data below shows the results of his poll.

   3, 2, 5, 5, 8, 4, 3, 4, 7, 2, 4, 8, 5, 10, 5

   a. Display the data in a line plot.
   b. Find the median of the data.
   c. Find the mode or modes of the data.
   d. Find the range of the data.

29. The school bus Nico rides is designed to carry 48 students. Thirteen students were already on the bus when Nico boarded the bus this morning. Eleven more students boarded the bus while Nico was riding to school. How many seats on the bus were empty when it arrived at school?

30. The number of students enrolled at five different elementary schools is shown below.

   341  307  462  289  420

   a. Estimate. Estimate the number of students altogether who attend the schools.
   b. Use your estimated total to find the approximate average number of students in each school.
• Multiplying Mixed Numbers

**Power Up**

**Power Up K**

**facts**

**mental math**

a. **Measurement:** The mass of a softball is about 200 grams. What is the approximate mass of 5 softballs?

b. **Measurement:** Brianna poured out 375 mL from the 1-liter bottle of water. How many mL were left in the bottle?

c. **Percent:** How much is 25% of $60?

d. **Percent:** How much is 25% less than $60?

e. **Percent:** How much is 25% more than $60?

f. **Number Sense:** A score is a set of 20. Two score is 40. Three score equals how many dozen?

g. **Calculation:** $5^2, -5, \times 3, + 3, \div 9, \times 4, -1, \div 3$

h. **Roman Numerals:** Write the current year in Roman numerals.

**problem solving**

Choose an appropriate problem-solving strategy to solve this problem. The word BOB has a horizontal line of symmetry because each of its letters has a horizontal line of symmetry.

Write “BOB” on a sheet of paper. Fold the upper half of the word along the line of symmetry. The lower half of the word should look like this: DUD

Place the paper against a reflective surface or mirror. Notice that the upper half of the word “reappears.”

Other words that have a horizontal line of symmetry are BED, BOOK, HE and HI. Try the activity again using one of these words. Explain how this “trick” works.
To multiply mixed numbers, we change the mixed numbers to improper fractions before we multiply.

First change the mixed numbers to improper fractions.

\[
\frac{5}{2} \times \frac{5}{3} = \frac{25}{6}
\]

Then multiply.

\[
\frac{25}{6} = 4\frac{1}{6}
\]

Then simplify.

Example 1

Multiply: \(\frac{1}{5} \times 4\frac{1}{2}\)

First we write the mixed number as an improper fraction. When both numbers are written as fractions, we multiply. We find that \(\frac{1}{5}\) of \(4\frac{1}{2}\) is \(\frac{9}{10}\).

**Justify** Can we use \(\frac{9}{10} \div \frac{9}{2}\) to check the answer? Why or why not?

\[
\frac{1}{5} \times \frac{9}{2} = \frac{9}{10}
\]

Example 2

Multiply: \(3 \times 2\frac{1}{3}\)

We write both numbers as improper fractions; then we multiply.

\[
3 \times 2\frac{1}{2} = \frac{3}{1} \times \frac{7}{3} = \frac{21}{3} = 7
\]

We simplified the result to find that the product is 7. We found our answer by multiplying. We find the same answer if we add:

\[
2\frac{1}{3} + 2\frac{1}{3} + 2\frac{1}{3} = 6\frac{3}{3} = 7
\]

Lesson Practice

Multiply:

a. \(1\frac{1}{2} \times 1\frac{3}{4}\)  
b. \(3\frac{1}{2} \times 1\frac{2}{3}\)  
c. \(3 \times 2\frac{1}{2}\)

d. \(4 \times 3\frac{2}{3}\)  
e. \(1\frac{1}{3} \times 2\frac{1}{3}\)  
f. \(\frac{1}{6} \times 2\frac{5}{6}\)
*1. Copy the decimals chart from Lesson 119.

2. a. Name this figure.
   b. How many faces does this figure have?
   c. How many vertices does this figure have?
   d. Which faces are congruent and parallel?

3. Write the following sentence using digits and symbols:
   *The sum of two and two equals the product of two and two.*

**4. Multiple Choice** Which of these is not equal to \( \frac{1}{2} \)?
   A 0.5  B 50%  C 0.50  D 0.05

5. Lily cares for a cat and a kitten. The cat weighs \( 7 \frac{3}{4} \) pounds. What is a reasonable estimate of the kitten’s weight if the cat and the kitten together weigh about 11 pounds? Explain why your estimate is reasonable.

6. Jillian can type 4 pages in 1 hour. At that rate, how long will it take her to type 100 pages?

7. In rectangle \(ABCD\), \(BC\) is twice the length of \(AB\). Segment \(AB\) is 3 inches long.
   a. What is the perimeter of the rectangle?
   b. What is the area of the rectangle?
   c. Name two pairs of parallel sides.
   d. Name two pairs of perpendicular sides.
8. Emilio is about to roll a standard number cube.
   a. What is the probability that he will get a prime number in one roll?
   b. What is the chance that he will not get a prime number in one roll?

9. A decagon has how many more sides than a pentagon?

10. What is the average of 2, 4, 6, and 8?

11. $QR$ equals $RS$. $ST$ is 5 cm. $RT$ is 7 cm. Find $QT$.

12. $38.248 + 7.5 + 37.23 + 15$

13. $6 - (1.49 - 75c)$

14. $2.4 \times 100$

15. $0.24 \times 0.12$

16. $25 \times 50$

17. $8 \sqrt{0.1000}$

18. $0.5 \sqrt{4.35}$

19. $12 \sqrt{1440}$

20. $\frac{3}{3} + \frac{7}{4}$

21. $\frac{3}{7} + \frac{1}{2}$

22. $\frac{6}{15} - \frac{1}{5}$

23. $\frac{4}{5} - \frac{1}{3}$

24. $\frac{1}{2} \times 3 \frac{1}{3}$

25. $4 \times 2 \frac{1}{2}$

26. a. What is the area of a bedroom that is 3 meters wide and 4.5 meters long?
   b. What is the perimeter?

27. What is the volume of a drawer that is 2 ft by 1.5 ft by 0.5 ft?
**28.** Refer to the figure at right to solve parts a–c.

(a) **Analyze** The perimeter of each small equilateral triangle is 6 inches. What is the perimeter of the large equilateral triangle?

b. The area of one small triangle is what percent of the area of the large triangle?

c. **Conclude** A sequence of triangle patterns is shown below. Draw the next triangle in the pattern on your paper. How many small triangles form the large triangle in your drawing?

\[ \triangle, \Delta \triangle, \Delta \Delta \triangle, \ldots \]

29. Four-hour admission to an outdoor water park costs $12.50 per person. Gary and three friends plan to visit the park. They have a discount coupon for $2 off each person's admission. What is the total cost of the tickets?

\[(12.50 \times 4) - (2 \times 4) = 42\]

30. **Justify** Wyatt estimated the quotient of 189 ÷ 5 to be about 40. Did Wyatt make a reasonable estimate? Explain why or why not.

**Early Finishers**

Ms. Valdez's car is being repaired. To get to and from school, her daughter Paula will walk a total of \(\frac{7}{8}\) of a mile each day for 9 days.

a. How far will Paula walk altogether? Estimate and then find the actual product.

b. Is your answer reasonable?

c. If Paula walked to school for 3 full school weeks, how far would she walk in all?
Focus on

• Tessellations

Archaeologists know that people have been using tiles to make mosaics and to decorate homes and other buildings since about 4000 B.C. The Romans called these tiles tesseliae, from which we get the word tessellation (tiling). A tessellation is the repeated use of shapes to fill a flat surface without gaps or overlaps. Below are some examples of tessellations. We say that the polygons in these figures tessellate; in other words, they tile a plane.

These tessellations are called regular tessellations because one regular polygon is used again and again to tile the plane. Although the same shape is used repeatedly in regular tessellations, the orientation of the shape may vary from tile to tile. In Figure 1, for example, we see that all the triangles are congruent, but that alternate triangles are rotated 180° (half of a turn).

Now look at a vertex in each figure and count the number of polygons that meet at the vertex. Notice that a certain number of polygons meet at each vertex in each tessellation.

1. How many triangles meet at each vertex in Figure 1?
2. How many squares meet at each vertex in Figure 2?

Only a few regular polygons tessellate. Here is an example of a regular polygon that does not tessellate:

We see that the regular pentagon on the left will not fit into the gap formed by the other pentagons. Therefore, a regular pentagon does not tessellate.
3. **Multiple Choice** Which of these regular polygons tessellates? Draw a tessellation that uses that polygon.

A  
B  
C  
D  

There are some combinations of regular polygons that tessellate. Below is an example of a tessellation that combines regular hexagons and equilateral triangles. A tiling composed of two or more regular polygons such as this is called a *semiregular tessellation*.

4. **Multiple Choice** Which two of these regular polygons could combine to tile a plane? Draw a picture that shows the tessellation.

A  
B  
C  
D  

Many polygons that are not regular polygons can tile a plane. In fact, every triangle can tile a plane, and every quadrilateral can tile a plane. Here is an example using each type of polygon:

**Triangle**  
**Quadrilateral**

Notice in both examples that the tiles are congruent, but that alternate tiles are rotated 180°.

---

**Activity 1**

*Triangle and Quadrilateral Tessellations*

Materials needed:
- Lesson Activity 45
- scissors
5. Carefully cut out the triangles on **Lesson Activity 45**. On your desk, arrange the triangles like tiles so that the vertices of six triangles meet at a point and the sides align without gaps or overlapping. Do not flip (reflect) the triangles to make them fit.

6. Carefully cut out the quadrilaterals on **Lesson Activity 45**. When tiling with quadrilaterals, arrange the quadrilaterals so that the vertices of four quadrilaterals meet at a point.

Some polygons that tessellate can be carefully altered and fitted together to form intricate tessellations. In the example below, we start with an equilateral triangle and alter one side by cutting out a piece of the triangle. Then we attach the cutout piece to another side of the triangle. If we make several congruent figures, we can fit them together to tile a surface.

In the next example, we start with a square. We alter one side of the square and then make the corresponding alteration to the opposite side. Then we alter a third side of the square and make the corresponding alteration to the remaining side. Congruent copies of the figure tessellate.
Creating Tessellations with Altered Figures

Materials needed:
- Lesson Activity 46
- ruler
- several sheets of unlined paper
- scissors
- glue or tape
- colored pencils or crayons (optional)

In this activity you will alter a triangle or a square and then use the resulting figure to create a tessellation. First choose one of the two shapes at the bottom of Lesson Activity 46. Trace that figure onto a blank sheet of paper, using a ruler to keep the sides of the traced figure straight. Then cut out the traced figure with scissors. Now follow the set of directions below that applies to the shape you chose.

Triangle

Step 1: Alter one side of the triangle by cutting a section from the shape. Be sure to cut out only one section. (Do not cut several pieces from the shape.)

Step 2: Tape the cutout section to another side of the figure. Use scissors to cut away excess tape.

Step 3: Trace the altered figure 8 to 12 times onto blank paper. You may color the figures you traced with colored pencils or crayons.

Step 4: Use scissors to cut out the traced figures.

Step 5: Fit the figures together to tile a portion of the box provided on Lesson Activity 46.

Step 6: Glue or tape the tiles into place.

Square

Step 1: Alter one side of the square by cutting a section from the shape. Be sure to cut out only one section. (Do not cut several pieces from the shape.)

Step 2: Tape the cutout section to the opposite side of the figure. Use scissors to cut away excess tape.

Step 3: Optional: Repeat Steps 1 and 2 to alter the remaining two sides of the figure.
**Step 4:** Trace the altered figure 8 to 12 times onto blank paper. You may color the figures you traced with colored pencils or crayons.

**Step 5:** Use scissors to cut out the traced figures.

**Step 6:** Fit the figures together to tile a portion of the box provided on Lesson Activity 46.

**Step 7:** Glue or tape the tiles into place.

**Investigate Further**

a. Find examples of tessellations in floor tiles at school or at home. Trace or copy the patterns, and bring them to class to display.

b. Search the Internet for information about tessellations. Share pictures and/or information you found with the rest of the class.

c. These figures have been sorted into a group by one common characteristic.

![Figure A](image1.png)  ![Figure B](image2.png)  ![Figure C](image3.png)  ![Figure D](image4.png)

Figures E and F do not belong in the group above.

![Figure E](image5.png)  ![Figure F](image6.png)

Draw a figure that belongs in the first group. Then explain how you found your answer and why your answer is reasonable.
Roman Numerals

Roman numerals were used by the ancient Romans to write numbers. Today Roman numerals are still used to number such things as book chapters, movie sequels, and Super Bowl games. We might also find Roman numerals on clocks and buildings.

Some Roman numerals are

- **I** which stands for 1
- **V** which stands for 5
- **X** which stands for 10

The Roman numeral system does not use place value. Instead, the values of the numerals are added or subtracted, depending on their position. For example,

II means 1 plus 1, which is 2. (II does not mean “11.”)

Below we list the Roman numerals for the numbers 1 through 20. Study the patterns.

- 1 = I
- 2 = II
- 3 = III
- 4 = IV
- 5 = V
- 6 = VI
- 7 = VII
- 8 = VIII
- 9 = IX
- 10 = X
- 11 = XI
- 12 = XII
- 13 = XIII
- 14 = XIV
- 15 = XV
- 16 = XVI
- 17 = XVII
- 18 = XVIII
- 19 = XIX
- 20 = XX

The multiples of 5 are 5, 10, 15, 20, ….. The numbers that are one less than these (4, 9, 14, 19, …) have Roman numerals that involve subtraction.
4 = IV  (“one less than five”)
9 = IX  (“one less than ten”)
14 = XIV  (ten plus “one less than five”)
19 = XIX  (ten plus “one less than ten”)

In each case where a smaller Roman numeral (I) precedes a larger Roman numeral (V or X), we subtract the smaller number from the larger number.

Example

a. Write XXVII in our number system.¹

b. Write 34 in Roman numerals.

a. We can break up the Roman numeral and see that it equals 2 tens plus 1 five plus 2 ones.

\[
\begin{align*}
XX & \quad V \\
20 + 5 + 2 & = 27
\end{align*}
\]

b. We think of 34 as “30 plus 4.”

\[
\begin{align*}
30 + 4 & \\
XXX & \quad IV
\end{align*}
\]

So the Roman numeral for 34 is XXXIV.

Lesson Practice

Write the Roman numerals for 1 to 39 in order.

¹ The modern world has adopted the Hindu-Arabic number system with the digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, and base ten place value. For simplicity, we refer to the Hindu-Arabic system as “our number system.”
• Roman Numerals Through Thousands

New Concept

We have practiced using these Roman numerals:

\[ \begin{align*}
\text{I} & \quad \text{V} \\
1 & \quad 5
\end{align*} \]

With these numerals we can write counting numbers up to XXXIX (39). To write larger numbers, we must use the Roman numerals L (50), C (100), D (500), and M (1000). The table below shows the different Roman numeral “digits” we have learned, as well as their respective values.

<table>
<thead>
<tr>
<th>Numeral</th>
<th>I</th>
<th>V</th>
<th>X</th>
<th>L</th>
<th>C</th>
<th>D</th>
<th>M</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value</td>
<td>1</td>
<td>5</td>
<td>10</td>
<td>50</td>
<td>100</td>
<td>500</td>
<td>1000</td>
</tr>
</tbody>
</table>

Example

Write each Roman numeral in our number system:

a. LXX
b. DCCL
c. XLIV
d. MMI

a. LXX is 50 + 10 + 10, which is 70.
b. DCCL is 500 + 100 + 100 + 50, which is 750.
c. XLIV is “10 less than 50” plus “1 less than 5,” that is, 40 + 4 = 44.
d. MMI is 1000 + 1000 + 1, which is 2001.

Lesson Practice

Write each Roman numeral in our number system:

a. CCCLXII
b. CCLXXXV
c. CD
d. XLVII
e. MMMCCLVI
f. MCMXCIX
### Acute Angle

<table>
<thead>
<tr>
<th>Term</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>acute angle</strong></td>
<td>An angle whose measure is more than 0° and less than 90°.</td>
</tr>
</tbody>
</table>

\[ \text{acute angle} \quad \text{not acute angles} \]

---

### Acute Triangle

<table>
<thead>
<tr>
<th>Term</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>acute triangle</strong></td>
<td>A triangle whose largest angle measures less than 90°.</td>
</tr>
</tbody>
</table>

\[ \text{acute triangle} \quad \text{not acute triangles} \]

---

### Addend

<table>
<thead>
<tr>
<th>Term</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>addend</strong></td>
<td>Any one of the numbers added in an addition problem.</td>
</tr>
</tbody>
</table>

\[ 7 + 3 = 10 \quad \text{The addends in this problem are 7 and 3.} \]

---

### Adjacent Sides

<table>
<thead>
<tr>
<th>Term</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>adjacent sides</strong></td>
<td>Sides that intersect.</td>
</tr>
</tbody>
</table>

---

### Algorithm

<table>
<thead>
<tr>
<th>Term</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>algorithm</strong></td>
<td>Any process for solving a mathematical problem.</td>
</tr>
</tbody>
</table>

\[ \text{In the addition algorithm we add the ones first, then the tens, and then the hundreds.} \]

---

### A.M.

<table>
<thead>
<tr>
<th>Term</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>a.m.</strong></td>
<td>The period of time from midnight to just before noon.</td>
</tr>
</tbody>
</table>

\[ \text{I get up at 7 a.m., which is 7 o'clock in the morning.} \]

---

### A.M.

<table>
<thead>
<tr>
<th>Term</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>a.m.</strong></td>
<td>The period of time from midnight to just before noon.</td>
</tr>
</tbody>
</table>

\[ \text{Me levanto a las 7 a.m., que son las 7 en punto de la mañana.} \]
<table>
<thead>
<tr>
<th><strong>angle</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>The opening that is formed when two lines, line segments, or rays intersect.</td>
</tr>
<tr>
<td>ángulo</td>
</tr>
<tr>
<td>Abertura que se forma cuando se intersecan dos rectas, rayos o segmentos de recta.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th><strong>area</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>The number of square units needed to cover a surface.</td>
</tr>
<tr>
<td>área</td>
</tr>
<tr>
<td>El número de unidades cuadradas que se necesitan para cubrir una superficie.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th><strong>arithmetic sequence</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>A sequence in which each term is found by adding a fixed amount to the previous term.</td>
</tr>
<tr>
<td>secuencia aritmética</td>
</tr>
<tr>
<td>Una secuencia en la que cada término se encuentra sumando una cantidad fija al término anterior.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th><strong>array</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>A rectangular arrangement of numbers or symbols in columns and rows.</td>
</tr>
<tr>
<td>matriz</td>
</tr>
<tr>
<td>Un arreglo rectangular de números o símbolos en columnas y filas.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th><strong>Associative Property of Addition</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>The grouping of addends does not affect their sum. In symbolic form, (a + (b + c) = (a + b) + c). Unlike addition, subtraction is not associative.</td>
</tr>
<tr>
<td>propiede asociativa de la suma</td>
</tr>
<tr>
<td>La agrupación de los sumandos no altera la suma. En forma simbólica, (a + (b + c) = (a + b) + c). A diferencia de la suma, la resta no es asociativa.</td>
</tr>
</tbody>
</table>
**Associative Property of Multiplication**

The grouping of factors does not affect their product. In symbolic form, \(a \times (b \times c) = (a \times b) \times c\). Unlike multiplication, division is not associative.

\[
(8 \times 4) \times 2 = 8 \times (4 \times 2) \quad (8 \div 4) \div 2 \neq 8 \div (4 \div 2)
\]

**Multiplication is associative.**  **Division is not associative.**

**average**

The number found when the sum of two or more numbers is divided by the number of addends in the sum; also called *mean*.

To find the average of the numbers 5, 6, and 10, first add.

\[5 + 6 + 10 = 21\]

Then, since there were three addends, divide the sum by 3.

\[21 \div 3 = 7\]

The average of 5, 6, and 10 is 7.

**bar graph**

A graph that uses rectangles (bars) to show numbers or measurements.

This bar graph shows how many rainy days there were in each of these four months.

**gráfica de barras**

Una gráfica que usa rectángulos (barras) para mostrar números o medidas.

Esta gráfica de barras muestra cuantos días lluviosos hubo en cada uno de estos cuatro meses.
**base**  
1. The lower number in an exponential expression.
   \[ \text{base} \rightarrow 5^3 \leftarrow \text{exponent} \]
   \(5^3\) means \(5 \times 5 \times 5\), and its value is 125.
2. A designated side or face of a geometric figure.

**base-ten system**  
A place-value system in which each place value is 10 times larger than the place value to its right.

*The decimal system is a base-ten system.*

**capacity**  
The amount of liquid a container can hold.

*Cups, gallons, and liters are units of capacity.*

**cardinal number(s)**  
The counting numbers \(1, 2, 3, 4, \ldots\)

**Celsius**  
A scale used on some thermometers to measure temperature.

*On the Celsius scale, water freezes at 0°C and boils at 100°C.*

**center**  
The point inside a circle from which all points on the circle are equally distant.

*The center of circle A is 2 inches from every point on the circle.*
**centigrade**

A metric system temperature scale with one hundred gradations, or degrees, between the freezing and boiling points of water.

*The Celsius scale is a centigrade scale.*

**centígrado**

Una escala de temperatura del sistema métrico con cien gradaciones o grados, entre el punto de ebullición y el de congelación del agua.

La escala de Celsius es una escala de centígrados.

**centimeter**

One hundredth of a meter.

*The width of your little finger is about one centimeter.*

**centímetro**

Una centésima de un metro.

El ancho de tu dedo meñique mide aproximadamente un centímetro.

**century**

A period of one hundred years.

*The years 2001–2100 make up one century.*

**siglo**

Un período de tiempo de cien años.

Los años del 2001 al 2100 forman un siglo.

**certain**

We say that an event is certain when the event’s probability is 1. This means the event will definitely occur.

**seguro**

Decimos que un evento es seguro cuando la probabilidad de que el evento ocurra es 1. Esto significa que el evento definitivamente va a ocurrir.

**chance**

A way of expressing the likelihood of an event; the probability of an event expressed as a percentage.

*The chance of rain is 20%. It is not likely to rain.*

*There is a 90% chance of snow. It is likely to snow.*

**posibilidad**

Modo de expresar la probabilidad de ocurrencia de un suceso; la probabilidad de un suceso expresada como porcentaje.

La posibilidad de lluvia es del 20%. Es poco probable que llueva.

Hay un 90% de posibilidad de nieve. Es muy probable que nieve.

**circle**

A closed, curved shape in which all points on the shape are the same distance from its center.

**círculo**

Una figura cerrada y curva en la cual todos los puntos están a la misma distancia de su centro.
**circle graph**

A graph made of a circle divided into sectors. Also called *pie chart* or *pie graph.*

![Chart showing shoe colors of students]

This *circle graph* displays data on students’ shoe color.

**gráfica circular**

Una gráfica circular está formada por un círculo dividido en sectores. También llamada *diagrama circular.*

Esta gráfica circular representa los datos del color de zapatos de los estudiantes.

**circumference**

The distance around a circle; the perimeter of a circle.

If the distance from point A around to point A is 3 inches, then the *circumference* of the circle is 3 inches.

**circunferencia**

La distancia alrededor de un círculo. Perímetro de un círculo.

Si la distancia desde el punto A alrededor del círculo hasta el punto A es 3 pulgadas, entonces la circunferencia del círculo mide 3 pulgadas.

**cluster**

A group of data points that are very close together.

**cúmulo**

Un grupo de puntos de datos que están muy cerca uno del otro.

**common denominators**

Denominators that are the same.

The fractions $\frac{2}{3}$ and $\frac{2}{5}$ have *common denominators.*

**denominadores comunes**

Denominadores que son iguales.

Las fracciones $\frac{2}{3}$ y $\frac{2}{5}$ tienen denominadores comunes.

**common fraction**

A fraction with whole-number terms.

<table>
<thead>
<tr>
<th>$\frac{1}{2}$</th>
<th>$\frac{5}{7}$</th>
<th>$\frac{3}{4}$</th>
<th>$\frac{1.2}{2.4}$</th>
<th>$\frac{3}{4.5}$</th>
<th>$\frac{2.5}{3}$</th>
</tr>
</thead>
</table>

*common fractions* not *common fractions*

**fracción común**

Una fracción con términos que son números enteros.
**common year**  
A year with 365 days; not a leap year.  
*The year 2000 is a leap year, but 2001 is a common year.*  
*In a common year February has 28 days. In a leap year it has 29 days.*

**Commutative Property of Addition**  
Changing the order of addends does not change their sum. In symbolic form, \( a + b = b + a \). Unlike addition, subtraction is not commutative.

\[
\begin{align*}
8 + 2 &= 2 + 8 \\
8 - 2 &\neq 2 - 8
\end{align*}
\]

*Addition is commutative.*  
*Subtraction is not commutative.*

**Commutative Property of Multiplication**  
Changing the order of factors does not change their product. In symbolic form, \( a \times b = b \times a \). Unlike multiplication, division is not commutative.

\[
\begin{align*}
8 \times 2 &= 2 \times 8 \\
8 \div 2 &\neq 2 \div 8
\end{align*}
\]

*Multiplication is commutative.*  
*Division is not commutative.*

**comparative graph**  
A method of displaying data, usually used to compare two or more related sets of data.

*This comparative graph compares how many sweaters were sold with how many T-shirts were sold in each of these six months.*

**gráfica comparativa**  
Un método para mostrar datos, usualmente usado para comparar dos o más conjuntos de datos relacionados.

*Esta gráfica comparativa compara cuántos suéteres se vendieron con cuántas camisetas se vendieron en cada uno de estos seis meses.*
**comparison symbol**  
A mathematical symbol used to compare numbers.  
*Comparison symbols* include the equal sign (=) and the “greater than/less than” symbols (>, or <).

**compatible numbers**  
Numbers that are close in value to the actual numbers and are easy to add, subtract, multiply, or divide mentally.

**composite number**  
A counting number greater than 1 that is divisible by a number other than itself and 1. Every composite number has three or more factors. Every composite number can be expressed as a product of two or more prime numbers.  
9 is divisible by 1, 3, and 9. It is composite.  
11 is divisible by 1 and 11. It is not composite.  

**cone**  
A three-dimensional solid with one curved surface and one flat, circular surface. The pointed end of a cone is its apex.

**congruent**  
Having the same size and shape.  
These polygons are congruent. They have the same size and shape.

**signo de comparación**  
Un símbolo matemático que se usa para comparar números.  
Los signos de comparación incluyen el signo de igual (=) y los signos “mayor que/menor que” (> o <).

**números compatibles**  
Números que están cerca en valor a los números originales y que son fáciles de sumar, restar, multiplicar o dividir mentalmente.

**número compuesto**  
Número de conteo mayor que 1, divisible entre algún otro número distinto de sí mismo y de 1. Cada número compuesto tiene tres o más divisores. Cada número compuesto puede ser expresado como el producto de dos o más números primos.  
9 es divisible entre 1, 3 y 9. Es compuesto.  
11 es divisible entre 1 y 11. No es compuesto.

**cono**  
Un sólido tridimensional de base circular y superficie curva. El extremo puntiagudo de un cono es su ápice.

**congruentes**  
Que tienen igual tamaño y forma.  
Estos polígonos son congruentes. Tienen igual tamaño y forma.
**continuous data**

Data that can be measured on a scale, such as length, elapsed time, temperature, and cost.

*Line graphs often display continuous data.*

**datos continuos**

Datos que se pueden medir en una escala, tal como longitud, tiempo transcurrido, temperatura y precio.

*Las gráficas lineales frecuentemente muestran datos continuos.*

**coordinate(s)**

1. A number used to locate a point on a number line.

![Coordinate](image)

*The coordinate of point A is −2.*

2. A pair of numbers used to locate a point on a coordinate plane.

![Coordinate Plane](image)

*The coordinates of point B are (2, 3). The x-coordinate is listed first, and the y-coordinate is listed second.*

**coordenada(s)**

1. Número que se utiliza para ubicar un punto sobre una recta numérica.

   *La coordenada del punto A es −2.*

2. Par ordenado de números que se utiliza para ubicar un punto sobre un plano coordenado.

   *Las coordenadas del punto B son (2, 3). La coordenada x se escribe primero, seguida de la coordenada y.*

**coordinate plane**

A grid on which any point can be identified by its distances from the x- and y-axes.

![Coordinate Plane](image)

*Point A is located at (−2, 2) on this coordinate plane.*

**plano coordenado**

Cuadrícula en que cualquier punto se puede identificar por sus distancias a los ejes x e y.

*El punto A está ubicado en la posición (−2, 2) sobre este plano coordenado.*
**counting numbers**

The numbers used to count; the numbers in this sequence: 1, 2, 3, 4, 5, 6, 7, 8, 9, ....

*The numbers 12 and 37 are counting numbers, but 0.98 and \( \frac{1}{2} \) are not.*

**números de conteo**

Números que se utilizan para contar; los números en esta secuencia: 1, 2, 3, 4, 5, 6, 7, 8, 9, ....

*Los números 12 y 37 son números de conteo pero 0.98 y \( \frac{1}{2} \) no lo son.*

**cube**

A three-dimensional solid with six square faces. Adjacent faces are perpendicular and opposite faces are parallel.

**cubo**

Un sólido tridimensional con seis caras cuadradas. Las caras adyacentes son perpendiculares y las caras opuestas son paralelas.

**cubic unit**

A cube with edges of designated length. Cubic units are used to measure volume.

*The shaded part is 1 cubic unit. The volume of the large cube is 8 cubic units.*

**unidad cúbica**

Un cubo con aristas de una longitud designada. Las unidades cúbicas se usan para medir volumen.

*La parte sombreada tiene 1 unidad cúbica. El volumen del cubo mayor es de 8 unidades cúbicas.*

**cylinder**

A three-dimensional solid with two circular bases that are opposite and parallel to each other.

**cilindro**

Un sólido tridimensional con dos bases circulares que son opuestas y paralelas entre sí.

**data**

(Singular: *datum*) Information gathered from observations or calculations.

82, 76, 95, 98, 97, 93

*These data are the average daily temperatures for one week in Utah.*

**datos**

Información reunida de observaciones o cálculos.

*Estos datos son el promedio diario de las temperaturas de una semana en Utah.*
<table>
<thead>
<tr>
<th>Term</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>decade</strong></td>
<td>A period of ten years.</td>
</tr>
<tr>
<td><strong>década</strong></td>
<td>Un periodo de diez años.</td>
</tr>
<tr>
<td><strong>decimal number</strong></td>
<td>A numeral that contains a decimal point.</td>
</tr>
<tr>
<td><strong>número decimal</strong></td>
<td>Número que contiene un punto decimal.</td>
</tr>
<tr>
<td><strong>decimal place(s)</strong></td>
<td>Places to the right of the decimal point.</td>
</tr>
<tr>
<td><strong>cifras decimales</strong></td>
<td>Números ubicados a la derecha del punto decimal.</td>
</tr>
<tr>
<td><strong>decimal point</strong></td>
<td>A symbol used to separate the ones place from the tenths place in decimal numbers (or dollars from cents in money).</td>
</tr>
<tr>
<td><strong>punto decimal</strong></td>
<td>Símbolo que se usa en números decimales para separar el lugar de las unidades del lugar de décimas.</td>
</tr>
<tr>
<td><strong>decimeter</strong></td>
<td>A metric unit of measurement equal to one tenth of a meter.</td>
</tr>
<tr>
<td><strong>decímetro</strong></td>
<td>Una unidad de medida métrica igual a una décima de un metro.</td>
</tr>
</tbody>
</table>
**degree (°)**

1. A unit for measuring temperature.

   ![Temperature Scale]

   Water boils. There are 100 degrees (100°) between the freezing and boiling points of water on the Celsius scale.

   Water freezes.

2. A unit for measuring angles.

   ![Angle Diagram]

   There are 90 degrees (90°) in a right angle. There are 360 degrees (360°) in a circle.

**grado (°)**

1. Unidad para medir temperaturas.
   
   Hay 100 grados de diferencia entre los puntos de ebullición y congelación del agua en la escala Celsius, o escala centígrada.

2. Unidad para medir ángulos.
   
   Hay 90 grados (90°) en un ángulo recto.
   
   Hay 360 grados (360°) en un círculo.

**denominator**

The bottom number of a fraction; the number that tells how many parts are in a whole.

![Fraction Diagram]

The denominator of the fraction is 4. There are 4 parts in the whole circle.

**denominador**

El número inferior de una fracción; el número que indica cuántas partes hay en un todo.


**diameter**

The distance across a circle through its center.

![Circle Diagram]

The diameter of this circle is 3 inches.

**diámetro**

Distancia entre dos puntos opuestos de un círculo a través de su centro.

El diámetro de este círculo mide 3 pulgadas.
**difference**

The result of subtraction.

\[ 12 - 8 = 4 \quad \text{The difference in this problem is 4.} \]

**digit**

Any of the symbols used to write numbers: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9.

\[ 12 \div 3 = 4 \quad 3 \overline{12} \quad \frac{12}{3} = 4 \quad \text{The dividend is 12 in each of these problems.} \]

**dimension**

The perpendicular measures of a figure.

Length and width are **dimensions** of a rectangle. Length, width, and height are **dimensions** of a rectangular prism.

**Distributive Property**

A number times the sum of two addends is equal to the sum of that same number times each individual addend:

\[ a \times (b + c) = (a \times b) + (a \times c) \]

\[ 8 \times (2 + 3) = (8 \times 2) + (8 \times 3) \]

Multiplication is **distributive** over addition.

**dividend**

A number that is divided.

\[ 6 \div 2 = 3 \quad 20 \div 3 = 6 R 2 \quad \text{The number 20 is **divisible** by 3, since 20 \div 3 has a remainder.} \]

**divisibility**

The ability for a number to be divided by another number without a remainder.

**divisible**

Able to be divided by a whole number without a remainder.

\[ \frac{5}{4} \quad \text{The number 20 is **divisible** by 4, since 20 \div 4 has no remainder.} \]

\[ \frac{6}{3} \quad \text{The number 20 is not **divisible** by 3, since 20 \div 3 has a remainder.} \]
divisible  Número que se puede dividir sin residuo entre un entero.

   El número 20 es divisible entre 4, ya que no tiene residuo.
   El número 20 no es divisible entre 3, ya que tiene residuo.

división  An operation that separates a number into a given number of equal parts or into a number of parts of a given size.

   \[ 21 \div 3 = 7 \]  We use division to separate 21 into 3 groups of 7.

divisor  A number by which another number is divided.

   \[ 12 \div 3 = 4 \]  The divisor is 3 in each of these problems.

edge  A line segment formed where two faces of a solid intersect.

   The arrow is pointing to one edge of this cube. A cube has 12 edges.

elapsed time  The difference between a starting time and an ending time.

   The race started at 6:30 p.m. and finished at 9:12 p.m. The elapsed time of the race was 2 hours 42 minutes.

dividendo  Número que divide a otro en una división.

   El divisor es 3 en cada una de estas operaciones.

E

endpoint(s)  The points at which a line segment ends.

   Points A and B are the endpoints of line segment AB.

extremo(s)  Punto donde termina un segmento de recta.

   Los puntos A y B son los extremos del segmento AB.
| **equation**  
(10) | A number sentence that uses the equal sign (\(=\)) to show that two quantities are equal.  
\[ x = 3 \quad 3 + 7 = 10 \quad 4 + 1 \quad x < 7 \]  
\[ \text{equations} \quad \text{not equations} \]  
**ecuación** | Enunciado de números que usa el símbolo de igualdad (\(=\)) para indicar que dos cantidades son iguales.  
\[ x = 3 \quad 3 + 7 = 10 \quad 4 + 1 \quad x < 7 \]  
**equations** | **not equations** |
| **equilateral triangle**  
(36) | A triangle in which all sides are the same length and all angles are the same measure.  
\[ \text{This is an equilateral triangle.} \]  
\[ \text{All of its sides are the same length.} \]  
\[ \text{All of its angles are the same measure.} \]  
\[ \text{triángulo equilátero} \] | Triángulo que tiene todos sus lados de la misma longitud y todos sus ángulos de la misma medida.  
\[ \text{Éste es un triángulo equilátero.} \]  
\[ \text{Todos sus lados tienen la misma longitud.} \]  
\[ \text{Todos sus ángulos tienen la misma medida.} \]  
**equivale a** | **no es equivalente a** |
| **equivalent fractions**  
(78) | Different fractions that name the same amount.  
\[ \frac{1}{2} \quad \frac{2}{4} \]  
\[ = \]  
\[ \frac{1}{2} \text{ and } \frac{2}{4} \text{ are equivalent fractions.} \]  
\[ \text{fracciones equivalentes} \] | Fracciones diferentes que representan la misma cantidad.  
\[ \frac{1}{2} \text{ y } \frac{2}{4} \text{ son fracciones equivalentes.} \]  
| **estimate**  
(33) | To find an approximate value.  
\[ \text{I estimate that the sum of 199 and 205 is about 400.} \]  
\[ \text{estimar} \] | Encontrar un valor aproximado.  
\[ \text{Puedo estimar que la suma de 199 más 205 es aproximadamente 400.} \]  
| **evaluate**  
(78) | To find the value of an expression.  
\[ \text{To evaluate } a + b \text{ for } a = 7 \text{ and } b = 13, \text{ we replace } a \text{ with 7 and } b \text{ with 13:} \]  
\[ 7 + 13 = 20 \]  
\[ \text{evaluar} \] | Calcular el valor de una expresión.  
\[ \text{Para evaluar } a + b, \text{ con } a = 7 \text{ y } b = 13, \text{ se reemplaza } a \text{ por } 7 \text{ y } b \text{ por 13:} \]  
\[ 7 + 13 = 20 \]  
| **even numbers**  
(2) | Numbers that can be divided by 2 without a remainder; the numbers in this sequence: 0, 2, 4, 6, 8, 10, ....  
\[ \text{Even numbers have } 0, 2, 4, 6, \text{ or 8 in the ones place.} \]  
\[ \text{números pares} \] | Números que se pueden dividir entre 2 sin residuo; los números en esta secuencia: 0, 2, 4, 6, 8, 10, ....  
\[ \text{Los números pares tienen } 0, 2, 4, 6 \text{ u } 8 \text{ en el lugar de las unidades.} \]  
|
**event**  
An outcome or group of outcomes in an experiment involving probability.

*The event of rolling a 4 with one roll of a standard number cube has a probability of $\frac{1}{6}$.***

**suceso**  
El resultado o grupo de resultados en un experimento que involucra probabilidad.

*El suceso de obtener un 4 al lanzar una vez un cubo de números tiene una probabilidad de $\frac{1}{6}$.***

**expanded form**  
A way of writing a number that shows the value of each digit.

*The expanded form of 234 is 200 + 30 + 4.*

**expanded notation**  
A way of writing a number as the sum of the products of the digits and the place values of the digits.

*In expanded notation 6753 is written as $(6 \times 1000) + (7 \times 100) + (5 \times 10) + (3 \times 1)$.*

**experiment**  
A test to find or illustrate a rule.

*Flipping a coin and selecting an object from a collection of objects are two experiments that involve probability.*

**exponent**  
The upper number in an exponential expression; it shows how many times the base is to be used as a factor.

$5^3$ means $5 \times 5 \times 5$, and its value is 125.

**exponential expression**  
An expression that indicates that the base is to be used as a factor the number of times shown by the exponent.

$4^3 = 4 \times 4 \times 4 = 64$

*The exponential expression $4^3$ uses 4 as a factor 3 times. Its value is 64.*
expression

A number, a letter, or a combination of both. Expressions do not include comparison symbols, such as an equal sign.

3n is an expression that can also be written as \( 3 \times n \).

expresión

Un número, una letra o una combinación de los dos. Las expresiones no incluyen símbolos de comparación, como el signo de igual.

3n es una expresión que también puede ser escrita como \( 3 \times n \).

F

face

A flat surface of a geometric solid.

The arrow is pointing to one face of the cube. A cube has six faces.

cara

Superficie plana de un cuerpo geométrico.

La flecha apunta a una cara del cubo. Un cubo tiene seis caras.

fact family

A group of three numbers related by addition and subtraction or by multiplication and division.

The numbers 3, 4, and 7 are a fact family. They make these four facts:

\[
\begin{align*}
3 + 4 &= 7 \\
4 + 3 &= 7 \\
7 - 3 &= 4 \\
7 - 4 &= 3
\end{align*}
\]

familia de operaciones

Grupo de tres números relacionados por sumas y restas o por multiplicaciones y divisiones.

Los números 3, 4 y 7 forman una familia de operaciones. Con ellos se pueden formar estas cuatro operaciones:

\[
\begin{align*}
3 + 4 &= 7 \\
4 + 3 &= 7 \\
7 - 3 &= 4 \\
7 - 4 &= 3
\end{align*}
\]

factor

1. Noun: Any one of the numbers multiplied in a multiplication problem.

\[
2 \times 3 = 6 \quad \text{The factors in this problem are 2 and 3.}
\]

2. Noun: A whole number that divides another whole number without a remainder.

The numbers 2 and 3 are factors of 6.

3. Verb: To write as a product of factors.

We can factor the number 6 by writing it as \( 2 \times 3 \).

factor (n); factorizar (v)

1. Nombre o sustantivo: Cualquiera de los números multiplicados en un problema de multiplicación.

\[
2 \times 3 = 6 \quad \text{Los factores en esta operación son el 2 y el 3.}
\]


3. Verbo: Escribir como producto de factores.

Se puede factorizar el número 6 escribiéndolo como el producto \( 2 \times 3 \).
| **Fahrenheit** | A scale used on some thermometers to measure temperature.  

*On the Fahrenheit scale, water freezes at 32°F and boils at 212°F.* |
| Fahrenheit | Escala que se usa en algunos termómetros para medir temperatura.  

*En la escala Fahrenheit, el agua se congela a 32°F y hiere a 212°F.* |

| **Fibonacci sequence** | A famous sequence in mathematics, which follows an addition pattern.  

1, 1, 2, 3, 5, 8, …  

*Each term equals the sum of the two terms before it.*  

1 + 1 = 2, 1 + 2 = 3, 2 + 3 = 5 … |
| fibonacci sequence | Una famosa secuencia matemática que sigue un patrón de suma.  

1, 1, 2, 3, 5, 8 …  

*Cada término es igual a la suma de los dos términos anteriores.*  

1 + 1 = 2, 1 + 2 = 3, 2 + 3 = 5 … |

| **fluid ounce** | A unit of liquid measurement in the customary system equal to one sixteenth of a pint. |
| fluid ounce | Una unidad de medida para líquidos en el sistema usual que es igual a un dieciseisavo de pinta. |

| **formula** | An expression or equation that describes a method for solving a certain type of problem. We often write formulas with letters that stand for complete words.  

*A formula for the perimeter of a rectangle is* $P = 2l + 2w$, *where* $P$ *stands for “perimeter,”* $l$ *stands for “length,” and* $w$ *stands for “width.”* |
| formula | Una expresión o ecuación que describe un método para resolver cierto tipo de problemas. Frecuentemente escribimos fórmulas con letras que representan palabras completas.  

*Una fórmula para el perímetro del rectángulo es* $P = 2l + 2w$, *donde* $P$ *representa “perímetro”,* $l$ *representa “longitud” y* $w$ *representa “ancho”.* |

| **fraction** | A number that names part of a whole.  

$\frac{1}{4}$ of the circle is shaded.  

$\frac{1}{4}$ is a fraction.  

fración | Número que representa una parte de un entero.  

$\frac{1}{4}$ del círculo está sombreado. $\frac{1}{4}$ es una fracción.
**frequency**  
*inv. 5*  
The number of times an event or outcome occurs.

### Purchased Lunch

<table>
<thead>
<tr>
<th>Number of Lunches</th>
<th>Tally</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>I</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>IIIII</td>
<td>4</td>
</tr>
<tr>
<td>3</td>
<td>IIII</td>
<td>7</td>
</tr>
<tr>
<td>4</td>
<td>IIIII</td>
<td>10</td>
</tr>
<tr>
<td>5</td>
<td>III</td>
<td>3</td>
</tr>
</tbody>
</table>

*This table shows the frequency of students that purchased lunch.*

**frecuencia**  
El número de veces que un suceso o resultado ocurre.

*Esta tabla muestra la frecuencia de los estudiantes que compraron almuerzo.*

### Race Results

<table>
<thead>
<tr>
<th>Laps Completed</th>
<th>Tally</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>I</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>IIIII</td>
<td>4</td>
</tr>
<tr>
<td>3</td>
<td>IIII</td>
<td>7</td>
</tr>
<tr>
<td>4</td>
<td>IIIII</td>
<td>10</td>
</tr>
<tr>
<td>5</td>
<td>III</td>
<td>3</td>
</tr>
</tbody>
</table>

*This frequency table summarizes the students’ performance at the race.*

**tabla de frecuencias**  
Una tabla que se utiliza para tabular y mostrar el número de veces que un suceso o resultado ocurre.

*Esta tabla de frecuencias resume el desempeño de los estudiantes en la competencia.*

### function table  
*inv. 4*  
A table that shows the relationship (or function) between related pairs of numbers.

<table>
<thead>
<tr>
<th>In</th>
<th>Out</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>6</td>
</tr>
<tr>
<td>4</td>
<td>8</td>
</tr>
<tr>
<td>7</td>
<td>14</td>
</tr>
</tbody>
</table>

*This function table uses the rule “multiply by two.”*
**tabla de función**
Una tabla que muestra la relación (o función) entre pares de números relacionados. 
*Esta tabla de función utiliza la regla “multiplicar por dos”.*

**G**

**geometric sequence** *(Inv. 4)*
A sequence in which each term is found by multiplying the previous term by a fixed amount.

\[ \times 3 \times 3 \times 3 \times 3 \]

\[ 1, \ 3, \ 9, \ 27, \ 81, \ldots \]

*We multiply a term by 3 to find the term that follows it in this geometric sequence.*

**progresión geométrica**
Una secuencia en la que cada término se encuentra multiplicando el término anterior por una cantidad fija.

\[ \times 3 \times 3 \times 3 \times 3 \]

\[ 1, \ 3, \ 9, \ 27, \ 81, \ldots \]

*Multiplicamos un término por 3 para encontrar el término que sigue en esta progresión geométrica.*

**geometric solid** *(83)*
A shape that takes up space.

**sólido geométrico**
Figura que ocupa un espacio.

**geometry** *(12)*
A major branch of mathematics that deals with shapes, sizes, and other properties of figures.

*Some of the figures we study in geometry are angles, circles, and polygons.*

**geometría**
Una rama principal de matemáticas que trata de las formas, tamaños y otras propiedades de figuras.

*Algunas de las figuras que se estudian en geometría son los ángulos, círculos y polígonos.*

**graph** *(Inv. 5)*
1. Noun: A diagram that shows data in an organized way. See also bar graph, circle graph, line graph, and pictograph.

   ![Graph example](image)

   **bar graph**

   **circle graph**

2. Verb: To draw a point, line, or curve on a coordinate plane.

   1. Nombre: Un diagrama que muestra datos de una forma organizada. Ver también gráfica de barras, gráfica circular, gráfica lineal y pictograma.

   2. Verbo: Dibujar un punto, línea o curva en un plano coordenado.
greatest common factor (GCF) (82)

The largest whole number that is a factor of two or more given numbers.

The factors of 20 are 1, 2, 4, 5, 10, and 20.
The factors of 30 are 1, 2, 3, 5, 6, 10, 15, and 30.
The common factors of 20 and 30 are 1, 2, 5, and 10.
The greatest common factor of 20 and 30 is 10.

máximo común divisor (MCD)

Es el mayor número entero que es factor de dos o más números.
Los factores de 20 son 1, 2, 4, 5, 10 y 20.
Los factores de 30 son 1, 2, 3, 5, 6, 10, 15 y 30.
El máximo común divisor de 20 y 30 es 10.

H

half (2)
One of two equal parts that together equal a whole.
mitad
Una de dos partes que juntas forman un entero.

hexagon (32)
A six-sided polygon.

hexágono
Un polígono de seis lados.

histogram (Inv. 7)
A method of displaying a range of data. A histogram is a special type of bar graph that displays data in intervals of equal size with no space between bars.

Student Reading Time

<table>
<thead>
<tr>
<th>Time (minutes)</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>21-28</td>
<td>2</td>
</tr>
<tr>
<td>29-36</td>
<td>4</td>
</tr>
<tr>
<td>37-44</td>
<td>6</td>
</tr>
<tr>
<td>45-52</td>
<td>8</td>
</tr>
<tr>
<td>53-60</td>
<td>2</td>
</tr>
</tbody>
</table>

This is a histogram.

histograma
Método para representar un conjunto de datos. Un histograma es un tipo especial de gráfica de barras que muestra los datos a intervalos de igual tamaño y de manera continua sin espacios entre las barras.

horizontal (12)
Side to side; perpendicular to vertical.

horizontal line
horizontal
no horizontal lines
oblique line
vertical line

Lado a lado; perpendicular a una vertical.
**horizontal axis**

The scale of a graph that runs from left to right.

*eje horizontal*

La escala de una gráfica que va de izquierda a derecha.

---

**icon**

A symbol used in a pictograph to represent data.

<table>
<thead>
<tr>
<th>Consumed by Matt in One Day</th>
</tr>
</thead>
<tbody>
<tr>
<td>Water</td>
</tr>
<tr>
<td>Soda</td>
</tr>
<tr>
<td>Milk</td>
</tr>
<tr>
<td>Juice</td>
</tr>
</tbody>
</table>

Key: 🥗 = 1 cup = 8 ounces

*Each icon in the pictograph represents 1 cup of liquid that was consumed.*

---

**Identity Property of Addition**

The sum of any number and 0 is equal to the initial number. In symbolic form, \(a + 0 = a\). The number 0 is referred to as the *additive identity*.

*The Identity Property of Addition is shown by this statement:*

\[
13 + 0 = 13
\]

*propiedad de identidad de la suma*

La suma de cualquier número mas 0 es igual al número inicial. En forma simbólica, \(a + 0 = a\). El 0 se conoce como *identidad aditiva*.

La *propiedad de identidad de la suma* se muestra en el siguiente enunciado:

\[
13 + 0 = 13
\]

---

**Identity Property of Multiplication**

The product of any number and 1 is equal to the initial number. In symbolic form, \(a \times 1 = a\). The number 1 is referred to as the *multiplicative identity*.

*The Identity Property of Multiplication is shown by this statement:*

\[
94 \times 1 = 94
\]

*propiedad de identidad de la multiplicación*

El producto de cualquier número por 1 es igual al número inicial. En forma simbólica, \(a \times 1 = a\). El número 1 se conoce como *identidad multiplicativa*.

La *propiedad de identidad de la multiplicación* se muestra en el siguiente enunciado:

\[
94 \times 1 = 94
\]
**impossible**

We say that an event is *impossible* when the event’s probability is 0. This means the event will definitely not occur.

**imposible**

Decimos que un suceso es *imposible* cuando la probabilidad de que el suceso ocurra es 0. Esto significa que el suceso definitivamente no ocurrirá.

**improper fraction**

A fraction with a numerator greater than or equal to the denominator.

\[
\frac{3}{4} \quad \frac{2}{2}
\]

*These fractions are improper fractions.*

**fracción impropia**

Fracción con el numerador igual o mayor que el denominador.

\[
\frac{3}{4} \quad \frac{2}{2}
\]

*Estas fracciones son fracciones impropias.*

**integers**

The set of counting numbers, their opposites, and zero; the members of the set \{..., –2, –1, 0, 1, 2, ...\}.

\[–57 \text{ and } 4 \text{ are integers. } \frac{15}{8} \text{ and } –0.98 \text{ are not integers.}\]

**enteros positivos, negativos y el cero**

Conjunto de números de conteo, sus opuestos y el cero; los elementos del conjunto \{..., –2, –1, 0, 1, 2, ...\}.

\[–57 \text{ y } 4 \text{ son enteros. } \frac{15}{8} \text{ y } –0.98 \text{ no son enteros.}\]

**International System of Units**

See *metric system.*

**Sistema internacional de unidades**

Ver sistema métrico.

**intersect**

To share a common point or points.

\[\text{These two lines intersect. They share the common point } M.\]

**intersecar**

Tener uno o más puntos en común.

*Estas dos rectas se intersecan. Tienen el punto común M.*

**intersecting lines**

Lines that cross.

**líneas que se intersecan**

Líneas que se cruzan.
**inverse operation(s)**

An operation that undoes another.

- $a + b - b = a$
- $a - b + b = a$
- $a \times b \div b = a \ (b \neq 0)$
- $a \div b \times b = a \ (b \neq 0)$
- $\sqrt{a^2} = a \ (a \geq 0)$
- $(\sqrt{a})^2 = a \ (a \geq 0)$

Addition is the **inverse operation** of subtraction. Multiplication and division are **inverse operations**.

**invert**

To switch the numerator and denominator of a fraction to form its reciprocal.

*If we invert the fraction $\frac{3}{4}$ we get $\frac{4}{3}$.*

**isosceles triangle**

A triangle with at least two sides of equal length and two angles of equal measure.

Two of the sides of this **isosceles triangle** have equal lengths. Two of the angles have equal measures.

**itinerary**

A type of schedule that lists locations and destinations together with departure and arrival times.

Mr. Jones’ Trip

<table>
<thead>
<tr>
<th>Arrival City</th>
<th>Time</th>
<th>Departure City</th>
<th>Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>Boston, MA</td>
<td>6:55 a.m.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>New York City, NY</td>
<td>7:40 a.m.</td>
<td>Portland, ME</td>
<td>9:28 a.m.</td>
</tr>
<tr>
<td>Portland, ME</td>
<td>10:51 a.m.</td>
<td>New York City, NY</td>
<td>12:42 p.m.</td>
</tr>
<tr>
<td>Boston, MA</td>
<td>1:37 p.m.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Un tipo de programa que enlista el lugar y el destino junto con el tiempo de salida y llegada.
kilometer  
A metric unit of length equal to 1000 meters.  
*One kilometer is approximately 0.62 mile.*

kilómetro  
Una unidad métrica de longitud igual a 1000 metros.  
Un *kilómetro* es aproximadamente 0.62 *milla*.

leap year  
A year with 366 days; not a common year.  
*A leap year occurs every year that is divisible by 4, except for century years that are not divisible by 400. Thus the years 1700, 1800, and 1900 are not leap years because they are not divisible by 400 but 2000 is a leap year.*

año bisiesto  
Un año con 366 días; no es un año común.  
Un *año bisiesto* ocurre cada año que es divisible entre 4, a excepción de siglos que no son divisibles entre 400. Por lo tanto, los años 1700, 1800 y 1900 no fueron años bisiesitos porque no son divisibles entre 400 pero el 2000 fue un año bisiesto.

least common multiple (LCM)  
The smallest whole number that is a multiple of two or more given numbers.

The multiples of 4 are 4, 8, 12, 16, 20, …

The multiples of 6 are 6, 12, 18, 24, 30, …

The *least common multiple* of 4 and 6 is 12.

mínimo común múltiplo (mcm)  
El menor número entero que es múltiplo común de dos o más números dados.

Los múltiplos de 4 son 4, 8, 12, 16, 20, …  
Los múltiplos de 6 son 6, 12, 18, 24, 30, …  
El *mínimo común múltiplo* de 4 y 6 es 12.

legend  
A notation on a map, graph, or diagram that describes the meaning of the symbols and/or the scale used.

The *legend* of this scale drawing shows that $\frac{1}{4}$ inch represents 5 feet.

length  
A measure of the distance between any two points.

*The length of this nail is 3 inches.*
**longitud**  
Una medida de la distancia entre dos puntos.  
*La longitud de este clavo es 3 pulgadas.*

**line**  
(12) A straight collection of points extending in opposite directions without end.

**recta**  
Sucesión de puntos que se extiende indefinidamente en ambas direcciones.

**line graph**  
(Inv. 5, Inv. 6) A graph that connects points to show how information changes over time.

![Line Graph Example](image)

This is a **line graph**.

**gráfica lineal**  
Una gráfica que conecta puntos que muestran cómo cambia la información con el tiempo.

**line of symmetry**  
(105) A line that divides a figure into two halves that are mirror images of each other. See also symmetry.

![Line of Symmetry Examples](image)

**eje de simetría**  
Recta que divide una figura en dos mitades, en la cual una mitad es la imagen espejo de la otra. Ver simetría.

**line plot**  
(Inv. 5) A method of plotting a set of numbers by placing a mark above a number on a number line each time it occurs in the set.

![Line Plot Example](image)

This is a **line plot** of the numbers 5, 8, 8, 10, 10, 11, 12, 12, 12, 12, 13, 14, 16, 17, 17, 18, and 19.

**diagrama de puntos**  
Método para representar un conjunto de números, que consiste en colocar una marca sobre un número de una recta numérica cada vez que dicho número ocurre en el conjunto.  
*Éste es un diagrama de puntos de los números 5, 8, 8, 10, 10, 11, 12, 12, 12, 13, 13, 14, 16, 17, 17, 18, y 19.*
**line segment**  
A part of a line with two distinct endpoints. 
\[ \overline{AB} \text{ is a line segment.} \]

**liter**  
A metric unit of capacity or volume.  
*An liter is a little more than a quart.*

**mass**  
The amount of matter an object contains. A kilogram is a metric unit of mass.
*The mass of a bowling ball would be the same on the moon as on Earth, even though the weight of the bowling ball would be different.*

**mean**  
See *average.*

**measure of central tendency**  
A value that describes a property of a list of data, such as the middle number of the list or the number that appears in the list most often. *See also mean, median, and mode.*

1, 3, 5, 6, 8, 9, 13

*The median of this set is 6. The median of a set is one measure of central tendency.*

822  *Saxon Math Intermediate 5*
**median**

The middle number (or the average of the two central numbers) of a list of data when the numbers are arranged in order from the least to the greatest.

1, 1, 2, 4, 5, 7, 9, 15, 24, 36, 44

In this list of data, 7 is the median.

**mediana**

Número de en medio (o el promedio de los dos números centrales) en una lista de datos, cuando los números se ordenan de menor a mayor.

1, 1, 2, 4, 5, 7, 9, 15, 24, 36, 44  En esta lista de datos, 7 es la mediana.

---

**meter**

The basic unit of length in the metric system.

Many classrooms are about 10 meters long and 10 meters wide.

**metro**

La unidad básica de longitud en el sistema métrico.

Muchos salones de clase miden aproximadamente 10 metros de largo por 10 metros de ancho.

---

**metric system**

An international system of measurement in which units are related by a power of ten. Also called the International System.

Centimeters and kilograms are units in the metric system.

**sistema métrico**

Un sistema internacional de medición en el cual las unidades de medida se relacionan por potencias de diez. También se le llama Sistema internacional.

Centímetros y kilogramos son unidades del sistema métrico.

---

**mill**

An amount of money equal to one thousandth of a dollar (one tenth of a penny).

The gasoline price of $3.199 per gallon equals $3.19 plus 9 mills.

**milésima**

Una cantidad de dinero igual a una milésima de un dólar (un décimo de un centavo).

El precio de $3.199 por galón es igual a $3.19 más 9 milésimas.

---

**millennium**

A period of one thousand years.

The years 2001–3000 make up one millennium.

**milenio**

Un período de mil años.

Los años 2001–3000 forman un milenio.

---

**millimeter**

A metric unit of length equal to one thousandth of a meter.

There are 1000 millimeters in 1 meter and 10 millimeters in one centimeter.

**milímetro**

Una unidad métrica de longitud que es igual a una milésima de un metro.

Hay 1000 milímetros en 1 metro y 10 milímetros en un centímetro.

---

**mixed number**

A number expressed as a whole number plus a fraction.

The mixed number $2\frac{1}{3}$ means “two and one third.”

**número mixto**

Número formado por un número entero y una fracción.

El número mixto $2\frac{1}{3}$ significa "dos y un tercio".
**mode**  
*(Inv. 5, 84)*  
The number or numbers that appear most often in a list of data.  
5, 12, 32, 5, 16, 5, 7, 12  
*In this list of data, the number 5 is the mode.*

**moda**  
Número o números que aparecen con más frecuencia en una lista de datos.  
5, 12, 32, 5, 16, 5, 7, 12  
*En esta lista de datos, el número 5 es la moda.*

**multiple**  
*(15, 29)*  
A product of a counting number and another number.  
The **multiples** of 3 include 3, 6, 9, and 12.  

**múltiplo**  
Producto de un número de conteo por otro número.  
Los **múltiplos** de 3 incluyen 3, 6, 9 y 12.

**multiplication table**  
*(15)*  
A table used to find the product of two numbers. The product of two numbers is found at the intersection of the row and the column for the two numbers.  

**tabla de multiplicación**  
Una tabla que se utiliza para encontrar el producto de dos números. El producto de dos números se encuentra en la intersección de la fila y la columna para los dos números.

**mutually exclusive**  
*(Inv. 7)*  
Categories are mutually exclusive if each data point can be placed in one, and only one, of the categories.  

**mutuamente excluyentes**  
Dos categorías son mutuamente excluyentes si cada punto de los datos puede ser colocado en una, y solo una, de las categorías.  

*When flipping one coin, the categories are “landing heads-up” and “landing tails-up.” One coin cannot land both heads-up and tails-up on the same toss. Thus, the categories “landing heads-up” and “landing tails-up” are mutually exclusive.*  

**negative numbers**  
*(12)*  
Numbers less than zero.  
–15 and –2.86 are **negative numbers**.  
19 and 0.74 are not **negative numbers**.

**números negativos**  
Los números menores que cero.  
–15 y –2.86 son **números negativos**.  
19 y 0.74 no son **números negativos**.

**number line**  
*(12)*  
A line for representing and graphing numbers. Each point on the line corresponds to a number.  

**recta numérica**  
Recta para representar y graficar números. Cada punto de la recta corresponde a un número.
**numeral**

(Appendix A)

A symbol or group of symbols that represents a number. 

4, 72, and \( \frac{1}{2} \) are examples of **numerals**.

“Four,” “seventy-two,” and “one half” are words that name numbers but are not **numerals**.

**numeral**

Símbolo, o grupo de símbolos numéricos, que representa un número.

4, 72 y \( \frac{1}{2} \) son ejemplos de **numerales**. “Cuatro”, “setenta y dos” y “un medio” son palabras que identifican números, pero no son **numerales**.

**numerator**

(Inv. 2)

The top number of a fraction; the number that tells how many parts of a whole are counted.

The **numerator** of the fraction is 1. One part of the whole circle is shaded.

**numerador**

El término superior de una fracción. El número que nos dice cuántas partes de un entero se cuentan.

El **numerador** de la fracción es 1. Una parte del círculo está sombreada.

**oblique**

(12, 31)

1. Slanted or sloping: not horizontal or vertical.

2. Lines in the same plane that are neither parallel nor perpendicular.

**obtuse angle**

(31)

An angle whose measure is more than 90° and less than 180°.

An **obtuse angle** is larger than both a right angle and an acute angle.

**ángulo obtuso**

Ángulo que mide más de 90° y menos de 180°.

Un **ángulo obtuso** es más grande que un ángulo recto y que un ángulo agudo.
obtuse triangle
A triangle whose largest angle measures more than 90° and less than 180°.

triángulo obtusángulo
Triángulo cuyo ángulo mayor mide más de 90° y menos de 180°.

octagon
A polygon with eight sides.

octágono
Un polígono con ocho lados.

odd numbers
Numbers that have a remainder of 1 when divided by 2; the numbers in this sequence: 1, 3, 5, 7, 9, 11, ....

números impares
Números que cuando se dividen entre 2 tienen residuo 1; los números en la secuencia: 1, 3, 5, 7, 9, 11, ....

operations of arithmetic
The four basic mathematical operations: addition, subtraction, multiplication, and division.

operaciones aritméticas
Las cuatro operaciones matemáticas básicas: suma, resta, multiplicación y división.

opposite sides
Sides that are across from each other.

lados opuestos
Lados que están uno enfrente del otro.

ordinal numbers
Numbers that describe position or order.

números ordinales
Números que describen orden o posición.
\textbf{origin}  
\textit{(Inv. 8)}  
1. The location of the number 0 on a number line.

\begin{center}
\includegraphics[width=0.5\textwidth]{number_line_origin}
\end{center}

\textit{origin} on a number line

2. The point (0,0) on a coordinate plane.

\begin{center}
\includegraphics[width=0.5\textwidth]{coordinate_plane_origin}
\end{center}

\textit{origin} on a coordinate plane

\textbf{origin}  
1. Posición del número 0 en una recta numérica.  
2. El punto (0, 0) en un plano coordenado.

\textbf{ounce}  
\textit{(85)}  
A unit of weight in the customary system. Also a measure of liquid capacity. See also \textit{fluid ounce}.

\textit{Sixteen ounces equals a pound. Sixteen fluid ounces equals a pint.}

\textbf{onza}  
Una unidad de peso en el sistema usual. También una medida de capacidad.  
\textit{Ver también onza líquida.}

\textit{Dieciséis onzas es igual a una libra. Dieciséis onzas líquidas es igual a una pinta.}

\textbf{outcome}  
\textit{(57)}  
Any possible result of an experiment.

\textit{When rolling a number cube, the possible outcomes are 1, 2, 3, 4, 5, and 6.}

\textbf{resultado}  
Cualquier resultado posible de un experimento.

\textit{Cuando se lanza un cubo de números los resultados posibles son 1, 2, 3, 4, 5 y 6.}

\textbf{outlier}  
\textit{(Inv. 5)}  
A number that is distant from most of the other numbers in a list of data.

\textit{In the data at right, the number 28 is an outlier because it is distant from the other numbers in the list.}

\textbf{valor extremo}  
Número en una lista de datos, que es mucho mayor o mucho menor que los demás números de la lista.

\textit{En los datos a la derecha, el número 28 es un valor extremo, porque su valor es mayor que el de los demás números de la lista.}

\textbf{P}  
\textbf{parallel lines}  
\textit{(31)}  
Lines that stay the same distance apart; lines that do not cross.

\begin{center}
\includegraphics[width=0.5\textwidth]{parallel_lines}
\end{center}

\textit{parallel lines}

\textbf{rectas paralelas}  
Rectas ubicadas en un mismo plano y que nunca se intersecan.
**parallelogram**  
A quadrilateral that has two pairs of parallel sides.

![Parallelograms](image)

**paralelogramo**  
Cuadrilátero que tiene dos pares de lados paralelos.

**parentheses**  
A pair of symbols used to separate parts of an expression so that those parts may be evaluated first: ( ).

In the expression $15 - (12 - 4)$, the *parentheses* indicate that $12 - 4$ should be calculated before subtracting the result from 15.

**paréntesis**  
Un par de símbolos que se utilizan para separar partes de una expresión para que esas partes puedan ser evaluadas primero: ( ).

En la expresión $15 - (12 - 4)$ los *paréntesis* indican que $12 - 4$ debe ser calculado antes de restar el resultado de 15.

**partial product**  
When multiplying using pencil and paper, a product resulting from multiplying one factor by one digit of the other factor. The final product is the sum of the shifted partial products.

\[
\begin{array}{c}
53 \\
\times 26 \\
\hline
318 \\
+ 106 \\
\hline
1378
\end{array}
\]

**producto parcial**  
Cuando se multiplica usando lápiz y papel, el producto resulta de multiplicar un factor por un dígito del otro factor. El producto final es la suma de los productos parciales desplazados.

**pentagon**  
A polygon with five sides.

**pentágono**  
Un polígono de cinco lados.

**percent**  
A fraction whose denominator of 100 is expressed as a percent sign (%).

\[
\frac{99}{100} = 99\% = 99 \textit{ percent}
\]

**porcentaje**  
Fracción cuyo denominador de 100 se expresa con un signo (%), que se lee *por ciento*.
**perfect square**  
The product when a whole number is multiplied by itself.  
*The number 9 is a perfect square because $3 \times 3 = 9$.*

**cuadrado perfecto**  
Producto cuando un número entero se multiplica por sí mismo.  
*El número 9 es un cuadrado perfecto, porque $3 \times 3 = 9$.*

**perimeter**  
The distance around a closed, flat shape.  
*The perimeter of this rectangle (from point A around to point A) is 32 inches.*

**perímetro**  
Distancia alrededor de una figura cerrada y plana.  
*El perímetro de este rectángulo (desde el punto A alrededor del rectángulo hasta el punto A) es 32 pulgadas.*

**permutation**  
One possible arrangement of a set of objects.  

\[
2 \quad 4 \quad 3 \quad 1
\]

*The arrangement above is one possible permutation of the numbers 1, 2, 3, and 4.*

**permutación**  
Un arreglo posible de un conjunto de objetos.  
*El arreglo anterior es una permutación posible de los números 1, 2, 3 y 4.*

**perpendicular lines**  
Two lines that intersect at right angles.  

**rectas perpendiculares**  
Dos rectas que se intersecan formando ángulos rectos.

**pictograph**  
A graph that uses symbols to represent data.  

<table>
<thead>
<tr>
<th>Stars We Saw</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tom</td>
</tr>
<tr>
<td>Bob</td>
</tr>
<tr>
<td>Sue</td>
</tr>
<tr>
<td>Ming</td>
</tr>
<tr>
<td>Juan</td>
</tr>
</tbody>
</table>

*This is a pictograph.  
It shows how many stars each person saw.*

**pictograma**  
Gráfica que utiliza símbolos para representar datos.  
*Éste es un pictograma. Muestra el número de estrellas que vio cada persona.*
pie chart

See circle graph.

diagrama circular

Vea gráfica circular.

place value

The value of a digit based on its position within a number.

341  **Place value** tells us that 4 in 341 is worth “4 tens.”

23  In addition problems, we align digits with the same

+    7  **place value**.

371

valor posicional

Valor de un dígito de acuerdo al lugar que ocupa en el número.

El **valor posicional** indica que el 4 en 341 vale “cuatro decenas”. En los problemas de suma y resta, se alinean los dígitos que tienen el mismo **valor posicional**.

plane

A flat surface that has no boundaries.

*The flat surface of a desk is part of a plane.*

plane

Superficie plana ilimitada.

*La superficie plana de un escritorio es parte de un plano.*

plane figure

A flat shape.

[Diagram of plane figures]

figura plana

Una figura plana.

p.m.

The period of time from noon to just before midnight.

*I go to bed at 9 p.m., which is 9 o’clock at night.*

p.m.

Período de tiempo desde el mediodía hasta justo la medianoche.

*Me voy a dormir a las 9 p.m. lo cual es las 9 en punto de la noche.*

point

An exact position.

[Dot labeled A]

This dot represents **point** A.

punto

Una posición exacta.

Esta marca representa el **punto** A.

polygon

A closed, flat shape with straight sides.

[Diagram of polygons and non-polygons]

polígono

Figura cerrada y plana que tiene lados rectos.
<table>
<thead>
<tr>
<th><strong>positive numbers</strong></th>
<th>Numbers greater than zero.</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.25 and 157 are <strong>positive numbers</strong>.</td>
<td></td>
</tr>
<tr>
<td>−40 and 0 are not <strong>positive numbers</strong>.</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th><strong>número positivos</strong></th>
<th>Números mayores que cero.</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.25 y 157 son <strong>números positivos</strong>.</td>
<td></td>
</tr>
<tr>
<td>−40 y 0 no son <strong>números positivos</strong>.</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th><strong>power</strong></th>
<th>1. The value of an exponential expression.</th>
</tr>
</thead>
<tbody>
<tr>
<td>16 is the fourth <strong>power</strong> of 2 because $2^4 = 16$.</td>
<td></td>
</tr>
<tr>
<td>2. An exponent.</td>
<td></td>
</tr>
<tr>
<td>The expression $2^4$ is read “two to the fourth <strong>power</strong>.”</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th><strong>potencia</strong></th>
<th>1. El valor de una expresión exponencial.</th>
</tr>
</thead>
<tbody>
<tr>
<td>16 es la cuarta <strong>potencia</strong> de 2, porque $2^4 = 16$.</td>
<td></td>
</tr>
<tr>
<td>2. Un exponente.</td>
<td></td>
</tr>
<tr>
<td>La expresión $2^4$ se lee “dos a la cuarta <strong>potencia</strong>.”</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th><strong>prime number</strong></th>
<th>A counting number greater than 1 whose only two factors are the number 1 and itself.</th>
</tr>
</thead>
<tbody>
<tr>
<td>7 is a <strong>prime number</strong>. Its only factors are 1 and 7.</td>
<td></td>
</tr>
<tr>
<td>10 is <strong>not a prime number</strong>. Its factors are 1, 2, 5, and 10.</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th><strong>número primo</strong></th>
<th>Número natural mayor que 1, cuyos dos únicos factores son el 1 y el propio número.</th>
</tr>
</thead>
<tbody>
<tr>
<td>7 es un <strong>número primo</strong>. Sus únicos factores son 1 y 7.</td>
<td></td>
</tr>
<tr>
<td>10 no es un <strong>número primo</strong>. Sus factores son 1, 2, 5 y 10.</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th><strong>prism</strong></th>
<th>A three-dimensional solid with two congruent bases.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>prisma</strong></td>
<td>Un sólido tridimensional con dos bases congruentes.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th><strong>probability</strong></th>
<th>A way of describing the likelihood of an event; the ratio of favorable outcomes to all possible outcomes.</th>
</tr>
</thead>
<tbody>
<tr>
<td>The <strong>probability</strong> of rolling a 3 with a standard number cube is $\frac{1}{6}$.</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th><strong>probabilidad</strong></th>
<th>Manera de describir la ocurrencia de un suceso; la razón de resultados favorables a todos los resultados posibles.</th>
</tr>
</thead>
<tbody>
<tr>
<td>La <strong>probabilidad</strong> de obtener 3 al lanzar un cubo estándar de números es $\frac{1}{6}$.</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th><strong>product</strong></th>
<th>The result of multiplication.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$5 \times 3 = 15$ The <strong>product</strong> of 5 and 3 is 15.</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th><strong>producto</strong></th>
<th>Resultado de una multiplicación.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$5 \times 3 = 15$ El <strong>producto</strong> de 5 por 3 es 15.</td>
<td></td>
</tr>
<tr>
<td><strong>proper fraction</strong></td>
<td>A fraction whose denominator is greater than its numerator.</td>
</tr>
<tr>
<td>---------------------</td>
<td>------------------------------------------------------------</td>
</tr>
<tr>
<td>(75)</td>
<td>$\frac{3}{4}$ is a <em>proper fraction</em>.</td>
</tr>
<tr>
<td></td>
<td>$\frac{4}{3}$ is not a <em>proper fraction</em>.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th><strong>fracción propia</strong></th>
<th>Una fracción cuyo denominador es mayor que su numerador.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\frac{3}{4}$ es una <em>fracción propia</em>.</td>
</tr>
<tr>
<td></td>
<td>$\frac{4}{3}$ no es una <em>fracción propia</em>.</td>
</tr>
</tbody>
</table>

**Property of Zero for Multiplication**

Zero times any number is zero. In symbolic form, $0 \times a = 0$.

*The Property of Zero for Multiplication* tells us that $89 \times 0 = 0$.

<table>
<thead>
<tr>
<th><strong>propiedad del cero en la multiplicación</strong></th>
<th>Cero multiplicado por cualquier número es cero. En forma simbólica, $0 \times a = 0$.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td><em>La propiedad del cero en la multiplicación</em> dice que $89 \times 0 = 0$.</td>
</tr>
</tbody>
</table>

**protractor**

A tool used to measure and draw angles.

<table>
<thead>
<tr>
<th><strong>transportador</strong></th>
<th>Instrumento que sirve para medir y trazar ángulos.</th>
</tr>
</thead>
</table>

**pyramid**

A three-dimensional solid with a polygon as its base and triangular faces that meet at a vertex.

<table>
<thead>
<tr>
<th><strong>pirámide</strong></th>
<th>Un sólido tridimensional con un polígono en su base y caras triangulares que se encuentran en un vértice.</th>
</tr>
</thead>
</table>

**quadrilateral**

Any four-sided polygon.

<table>
<thead>
<tr>
<th><strong>cuadrilátero</strong></th>
<th>Polígono de cuatro lados. Cada uno de estos polígonos tiene 4 lados. Todos son <em>cuadriláteros</em>.</th>
</tr>
</thead>
</table>
**quotient** (20) The result of division.

\[
\begin{array}{c}
12 \div 3 = 4 \\
3 \overline{)12} \\
\frac{12}{3} = 4
\end{array}
\]

The quotient is 4 in each of these problems.

**cociente** Resultado de una división.

*El cociente es 4 en cada una de estas operaciones.*

---

**radius** (Plural: *radii*) The distance from the center of a circle to a point on the circle.

![Circle with radius labeled 2 in.](image)

*The radius of this circle is 2 inches.*

**radio** Distancia desde el centro de un círculo hasta un punto del círculo.

*El radio de este círculo es 2 pulgadas.*

---

**range** (Inv. 5, 84) The difference between the largest number and the smallest number in a list.

\[
5, 17, 12, 34, 29, 13
\]

To calculate the range of this list, we subtract the smallest number from the largest number. The range of this list is 29.

**intervalo** Diferencia entre el número mayor y el número menor de una lista.

*Para calcular el intervalo de esta lista, se resta el número menor del número mayor. El intervalo de esta lista es 29.*

---

**ratio** (97) A comparison of two numbers by division.

There are 3 triangles and 5 stars. The ratio of triangles to stars is “three to five,” or \(\frac{3}{5}\).

**razón** Comparación de dos números por división.

*Hay 3 triángulos y 5 estrellas. La razón de triángulos a estrellas es “tres a cinco” ó \(\frac{3}{5}\).*

---

**ray** (12) A part of a line that begins at a point and continues without end in one direction.

![Ray from A to B](image)

*Ray AB (AB)*

**rayo** Parte de una recta que empieza en un punto y continúa indefinidamente en una dirección.
**reciprocals**
Two numbers whose product is 1.

\[
\frac{3}{4} \times \frac{4}{3} = \frac{12}{12} = 1
\]

Thus, the fractions \(\frac{3}{4}\) and \(\frac{4}{3}\) are reciprocals. The reciprocal of \(\frac{3}{4}\) is \(\frac{4}{3}\).

**rectángulo**
Cuadrilátero que tiene cuatro ángulos rectos.

**rectangle**
A quadrilateral that has four right angles.

**rectangular solid**
A three-dimensional solid having six rectangular faces. Adjacent faces are perpendicular and opposite faces are parallel.

**reduce**
To rewrite a fraction in lowest terms.

*If we reduce the fraction \(\frac{9}{12}\), we get \(\frac{3}{4}\).*

**reflexión**
Inversión de una figura para lograr una imagen especular; imagen reflejada en un espejo.

**reflection**
Flipping a figure to produce a mirror image.

**reflective symmetry**
A figure has reflective symmetry if it can be divided into two halves that are mirror images of each other. See also line of symmetry.

These figures have reflective symmetry. These figures do not have reflective symmetry.

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Cuadrilátero que tiene cuatro ángulos rectos.

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Flipping a figure to produce a mirror image.

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A figure has reflective symmetry if it can be divided into two halves that are mirror images of each other. See also line of symmetry.

These figures have reflective symmetry. These figures do not have reflective symmetry.

**recíprocos**
Dos números cuyo producto es igual a 1.

Las fracciones \(\frac{3}{4}\) y \(\frac{4}{3}\) son recíprocas. El recíproco de \(\frac{3}{4}\) es \(\frac{4}{3}\).
regular polygon

A polygon in which all sides have equal lengths and all angles have equal measures.

polígono regular

Polígono en el cual todos los lados tienen la misma longitud y todos los ángulos tienen la misma medida.

relative frequency table

A frequency table in which the frequencies for all categories are displayed as the numerator of a fraction with the total number of outcomes as the denominator.

Outcome | Tally | Relative Frequency
--- | --- | ---
1 | 1 | 17/50
2 | 2 | 28/50
3 | 5 | 5/20

This relative frequency table shows data obtained by spinning the spinner at left 50 times.

tabla de frecuencias relativas

Una tabla de frecuencias en donde las frecuencias para todas las categorías se muestran como el numerador de una fracción con el número total de resultados como el denominador.

Esta tabla de frecuencias relativas muestra datos obtenidos al girar la rueda hacia la izquierda 50 veces.

remainder

An amount left after division.

When 15 is divided by 2, there is a remainder of 1.

residuo

Cantidad que queda después de dividir.

Cuando se divide 15 entre 2, queda residuo 1.

rhombus

A parallelogram with all four sides of equal length.

rombo

Paralelogramo con sus cuatro lados de igual longitud.
right angle  \((31)\)
An angle that forms a square corner and measures 90°. It is often marked with a small square.

A right angle is larger than an acute angle and smaller than an obtuse angle.

right triangle  \((36)\)
A triangle whose largest angle measures 90°.

Roman numerals  \((\text{Appendix A})\)
Symbols used by the ancient Romans to write numbers.

Roman numerals for 3 is III.

The Roman numeral for 13 is XIII.

rotation  \((\text{Inv. 8})\)
Turning a figure about a specified point called the center of rotation.

rotation

A figure has rotational symmetry if it can be rotated less than a full turn and appear in its original orientation.

These figures have rotational symmetry.

These figures do not have rotational symmetry.

rotational symmetry  \((105)\)

simetría rotacional
Una figura tiene simetría rotacional cuando no requiere de una rotación completa para que la figura aparezca en la misma posición en que comenzó la rotación.
round number (33)  A close number to the given number. Rounding a number can help us estimate.

número redondeado  Un número cercano al número dado. Redondear un número nos ayuda a estimar.

scale (27, Inv. 11)  
1. A type of number line used for measuring.

![cm scale](image)

The distance between each mark on this ruler’s scale is 1 centimeter.

2. A ratio that shows the relationship between a scale model and the actual object.

If a model airplane is \( \frac{1}{24} \) the size of the actual airplane, the scale of the model is 1 to 24.

escala  
1. Un tipo de recta numérica que se utiliza para hacer mediciones. 
La distancia entre cada marca en la escala de esta regla es 1 centímetro.

2. Una razón que nos muestra la relación entre un modelo a escala y el objeto actual. 
Si el modelo de un avión es \( \frac{1}{24} \) del tamaño del avión real, la escala del modelo es 1 a 24.

scale drawing (Inv. 11)  A two-dimensional representation of a larger or smaller object.

Blueprints and maps are examples of scale drawings.

dibujo a escala  Representación bidimensional de un objeto más grande o más pequeño.
Los planos y los mapas son ejemplos de dibujos a escala.

scale model (Inv. 11)  A three-dimensional representation of a larger or smaller object.

Globes and model airplanes are examples of scale models.

modelo a escala  Una representación tridimensional de un objeto más pequeño o más grande.
Los globos terrestres y aviones de juguete son ejemplos de modelos a escala.

scalene triangle (36)  A triangle with three sides of different lengths.

All three sides of this scalene triangle have different lengths.

triángulo escaleno  Triángulo con todos sus lados de diferente longitud.
Los tres lados de este triángulo escaleno tienen diferente longitud.
**schedule**
A list of events organized by the times at which they are planned to occur.

<table>
<thead>
<tr>
<th>Time</th>
<th>Activity</th>
</tr>
</thead>
<tbody>
<tr>
<td>8:15 a.m.</td>
<td>Homeroom</td>
</tr>
<tr>
<td>9:00 a.m.</td>
<td>Science</td>
</tr>
<tr>
<td>10:15 a.m.</td>
<td>Reading</td>
</tr>
<tr>
<td>11:30 a.m.</td>
<td>Lunch and recess</td>
</tr>
<tr>
<td>12:15 p.m.</td>
<td>Math</td>
</tr>
<tr>
<td>1:30 p.m.</td>
<td>English</td>
</tr>
<tr>
<td>2:45 p.m.</td>
<td>Art and music</td>
</tr>
<tr>
<td>3:30 p.m.</td>
<td>End of school</td>
</tr>
</tbody>
</table>

**sector**
A region bordered by part of a circle and two radii.

This circle is divided into 3 sectors. One sector of the circle is shaded.

**segment**
See line segment.

**sequence**
A list of numbers arranged according to a certain rule.

The numbers 2, 4, 6, 8, ... form a sequence. The rule is "count up by twos."

**short division**
A form of division that differs from long division. In short division we keep track of some numbers in our head.

**side**
A line segment that is part of a polygon.

The arrow is pointing to one side. This pentagon has 5 sides.
**lado**  Segmento de recta que forma parte de un polígono. La flecha apunta hacia uno de los lados. Este pentágono tiene 5 lados.

**similar**  Having the same shape but not necessarily the same size. Dimensions of similar figures are proportional.

\[ \triangle ABC \text{ and } \triangle DEF \text{ are similar. They have the same shape, but not the same size.} \]

**simplest form**  The form of a fraction when it is reduced to lowest terms.

**solid**  See geometric solid.

**sphere**  A round geometric solid having every point on its surface at an equal distance from its center.

**spread**  A value that describes how the data in a set are distributed. See also range.

5, 12, 3, 20, 15

The range of this set is 17. Range, which is the difference between the greatest and least numbers, is one measure of the spread of data.
square  1. A rectangle with all four sides of equal length.

   All four sides of this square are 12 mm long.

2. The product of a number and itself.

   The square of 4 is 16.

   cuadrado  1. Paralelogramo que tiene cuatro lados de igual longitud.
   Los cuatro lados de este cuadrado miden 12 mm.

   2. El producto de un número por sí mismo.
   El cuadrado de 4 es 16.

square centimeter  A measure of area equal to that of a square with 1-centimeter sides.

   centímetro cuadrado Medida de un área igual a la de un cuadrado con lados de 1 centímetro.

square inch  A measure of area equal to that of a square with 1-inch sides.

   pulgada cuadrada Medida de un área igual a la de un cuadrado con lados de 1 pulgada.

square root  One of two equal factors of a number. The symbol for the principal, or positive, square root of a number is \( \sqrt{\cdot} \).

   A square root of 49 is 7 because \( 7 \times 7 = 49 \).

   \[ \sqrt{49} = 7 \]

   raíz cuadrada Uno de dos factores iguales de un número. El símbolo de la raíz cuadrada principal, o positiva, de un número es \( \sqrt{\cdot} \).

   La raíz cuadrada de 49 es 7, porque \( 7 \times 7 = 49 \).

statistics  A branch of mathematics that deals with the collection, analysis, organization, and display of numerical data.

   Some activities performed in statistics are taking surveys and organizing data.

   estadística Una rama de las matemáticas que trata con la recolección, el análisis, la organización y la exhibición de los datos numéricos.

   Algunas actividades que se llevan a cabo en estadística son hacer encuestas y organizar datos.
**stem-and-leaf plot**  
A method of graphing a collection of numbers by placing the “stem” digits (or initial digits) in one column and the “leaf” digits (or remaining digits) out to the right.

<table>
<thead>
<tr>
<th>Stem</th>
<th>Leaf</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>1 3 5 6 6 8</td>
</tr>
<tr>
<td>3</td>
<td>0 0 2 2 4 5 6 6 8 9</td>
</tr>
<tr>
<td>4</td>
<td>0 0 1 1 1 2 3 3 5 7 7 8</td>
</tr>
<tr>
<td>5</td>
<td>0 1 1 2 3 5 8</td>
</tr>
</tbody>
</table>

*In this stem-and-leaf plot, 3|2 represents 32.*

**diagrama de hoja y tallos**  
Un método para graficar una colección de números colocando los dígitos del “tallo” (o dígitos iniciales) en una columna y los dígitos de las “hojas” (o dígitos restantes) hacia la derecha.

*En este diagrama de tallo y hojas, 3|2 representa 32.*

**straight angle**  
An angle that measures 180° and thus forms a straight line.

*Angle ABD is a straight angle.* Angles ABC and CBD are not straight angles.

**sum**  
The result of addition.

7 + 6 = 13  
The sum of 7 and 6 is 13.

**tally mark**  
A small mark used to help keep track of a count.

*I used tally marks to count cars.*

*I counted seven cars.*

**term**  
1. A number in a sequence.

1, 3, 5, 7, 9, 11, …

*Each number in this sequence is a term.*

2. A number that serves as a numerator or denominator of a fraction.

\[
\frac{5}{6}\]

término  
1. Un número de una secuencia.

1, 3, 5, 7, 9, 11, …

*Cada número de esta secuencia es un término.*

2. Número que se usa como numerador o denominador en una fracción.
**tessellation** *(Inv. 12)*  
The repeated use of shapes to fill a flat surface without gaps or overlaps.

**mosaico**  
El uso repetido de figuras para llenar una superficie plana sin dejar huecos ni superposiciones.

**tick mark** *(12)*  
Mark dividing a number line into smaller portions.

**marca de un punto**  
Marca que divide una recta número en partes más pequeñas.

**transformation** *(Inv. 8)*  
Changing a figure’s position through rotation, reflection, or translation.

![](image)

**Translations**

<table>
<thead>
<tr>
<th>Movement</th>
<th>Name</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flip</td>
<td>Reflection</td>
</tr>
<tr>
<td>Slide</td>
<td>Translation</td>
</tr>
<tr>
<td>Turn</td>
<td>Rotation</td>
</tr>
</tbody>
</table>

**transformación**  
Cambio en la posición de una figura por medio de una rotación, reflexión o traslación.

**translation** *(Inv. 8)*  
Sliding a figure from one position to another without turning or flipping the figure.

**traslación**  
Deslizamiento de una figura de una posición a otra, sin rotar ni voltear la figura.

**trapezium** *(45)*  
A quadrilateral with no parallel sides.

**trapezoide**  
Cuadrilátero que no tiene lados paralelos.
<table>
<thead>
<tr>
<th>Glossary Term</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>trapezoid</strong></td>
<td>A quadrilateral with exactly one pair of parallel sides.</td>
</tr>
<tr>
<td><strong>trapezoids</strong></td>
<td>Cuadrilátero que tiene exactamente un par de lados paralelos.</td>
</tr>
<tr>
<td><strong>triangle</strong></td>
<td>A polygon with three sides and three angles.</td>
</tr>
<tr>
<td><strong>triángulo</strong></td>
<td>Un polígono con tres lados y tres ángulos.</td>
</tr>
<tr>
<td><strong>triangular numbers</strong></td>
<td>Numbers that can be represented by objects arranged in a triangular pattern.</td>
</tr>
</tbody>
</table>

**Triangular numbers** include all the numbers in this sequence:

3, 6, 10, 15, 21, …

<table>
<thead>
<tr>
<th>Glossary Term</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>U.S. Customary System</strong></td>
<td>A system of measurement used almost exclusively in the United States.</td>
</tr>
<tr>
<td><strong>Sistema usual de EE.UU.</strong></td>
<td>Sistema de medición que se usa casi exclusivamente en EE.UU.</td>
</tr>
</tbody>
</table>

**Pounds, quarts, and feet** are units in the **U.S. Customary System**.

**Libras, cuartos y pies** son unidades del **Sistema usual de EE.UU.**
A diagram made of circles used to display data.

This Venn diagram shows data on students’ pets. Three students do not have a cat or a dog. Seven students have a dog, but not a cat. Six students have a cat, but not a dog. And five students have both a dog and a cat.

This diagram de Venn muestra los datos de las mascotas de estudiantes. Tres estudiantes no tienen gato ni perro. Siete estudiantes tienen perro pero no gato. Seis estudiantes tienen gato pero no perro. Y cinco estudiantes tienen tanto gato como perro.

(Plural: vertices) A point of an angle, polygon, or solid where two or more lines, rays, or line segments meet.

The arrow is pointing to one vertex of this cube. A cube has eight vertices.

Un diagrama que utiliza círculos para mostrar datos.

Este diagrama de Venn muestra los datos de las mascotas de estudiantes. Tres estudiantes no tienen gato ni perro. Siete estudiantes tienen perro pero no gato. Seis estudiantes tienen gato pero no perro. Y cinco estudiantes tienen tanto gato como perro.

Upright; perpendicular to horizontal.

Perpendicular a la horizontal.

The scale of a graph that runs from top to bottom.

La escala de una gráfica que corre de arriba hacia abajo.
volume
(103)
The amount of space a geometric solid occupies. Volume is measured in cubic units.

This rectangular prism is 3 units wide, 3 units high, and 4 units deep. Its volume is \(3 \cdot 3 \cdot 4 = 36\) cubic units.

volumen
Cantidad de espacio ocupado por un sólido geométrico. El volumen se mide en unidades cúbicas.

Este prisma rectangular tiene 3 unidades de ancho, 3 unidades de altura y 4 unidades de profundidad. Su volumen es \(3 \cdot 3 \cdot 4 = 36\) unidades cúbicas.

W

weight
(77)
The measure of the force of gravity on an object. Units of weight in the customary system include ounces, pounds, and tons.

The weight of a bowling ball would be less on the moon than on Earth because the force of gravity is weaker on the moon.

peso
La medida de la fuerza de gravedad sobre un objeto. Las unidades de peso en el sistema usual incluyen onzas, libras y toneladas.

El peso de una bola de boliche es menor en la Luna que en la Tierra porque la fuerza de gravedad es menor en la Luna.

whole number(s)
(2)
All the numbers in this sequence: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, ....

The number 35 is a whole number, but \(35 \frac{1}{2}\) and 4.2 are not. Whole numbers are the counting numbers and zero.

números enteros
Todos los números en esta secuencia: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 ....

El número 35 es un número entero pero \(35 \frac{1}{2}\) y 4.2 no lo son. Los números enteros son los números de conteo y el cero.

X

x-axis
(Inv. 8)
The horizontal number line of a coordinate plane.

eje de las x
Recta numérica horizontal en un plano coordenado.
**y-axis**  
The vertical number line of a coordinate plane.

**eje de las y**  
La recta numérica vertical en un plano coordenado.
### Symbols

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Meaning</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>△</td>
<td>Triangle</td>
<td>△ABC</td>
</tr>
<tr>
<td>∠</td>
<td>Angle</td>
<td>∠ABC</td>
</tr>
<tr>
<td>→</td>
<td>Ray</td>
<td>→AB</td>
</tr>
<tr>
<td>↔</td>
<td>Line</td>
<td>↔AB</td>
</tr>
<tr>
<td>—</td>
<td>Line segment</td>
<td>AB</td>
</tr>
<tr>
<td>⊥</td>
<td>Perpendicular to</td>
<td>AB ⊥ BC</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>&lt;</td>
<td>Less than</td>
<td>2 &lt; 3</td>
</tr>
<tr>
<td>&gt;</td>
<td>Greater than</td>
<td>3 &gt; 2</td>
</tr>
<tr>
<td>=</td>
<td>Equal to</td>
<td>2 = 2</td>
</tr>
<tr>
<td>°F</td>
<td>Degrees Fahrenheit</td>
<td>100°F</td>
</tr>
<tr>
<td>°C</td>
<td>Degrees Celsius</td>
<td>32°C</td>
</tr>
<tr>
<td>⊥</td>
<td>Right angle (90° angle)</td>
<td>90°</td>
</tr>
<tr>
<td>…</td>
<td>And so on</td>
<td>1, 2, 3,…</td>
</tr>
<tr>
<td>×</td>
<td>Multiply</td>
<td>9 × 3</td>
</tr>
<tr>
<td>·</td>
<td>Multiply</td>
<td>3 · 3 = 9</td>
</tr>
<tr>
<td>÷</td>
<td>Divide</td>
<td>9 ÷ 3</td>
</tr>
<tr>
<td>+</td>
<td>Add</td>
<td>9 + 3</td>
</tr>
<tr>
<td>−</td>
<td>Subtract</td>
<td>9 − 3</td>
</tr>
<tr>
<td>)⁻</td>
<td>Divided into</td>
<td>3)9</td>
</tr>
<tr>
<td>R or r</td>
<td>Remainder</td>
<td>3 R 2</td>
</tr>
<tr>
<td>%</td>
<td>Percent</td>
<td>50%</td>
</tr>
<tr>
<td>x²</td>
<td>“x” squared (times itself)</td>
<td>3² = 3 × 3 = 9</td>
</tr>
<tr>
<td>x³</td>
<td>“x” cubed</td>
<td>3³ = 3 × 3 × 3 = 27</td>
</tr>
<tr>
<td>√⁻</td>
<td>Square root</td>
<td>√9 = 3 because 3 × 3 = 9.</td>
</tr>
</tbody>
</table>

### Abbreviations

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>ft</td>
<td>Foot</td>
</tr>
<tr>
<td>in.</td>
<td>Inch</td>
</tr>
<tr>
<td>yd</td>
<td>Yard</td>
</tr>
<tr>
<td>mi</td>
<td>Mile</td>
</tr>
<tr>
<td>m</td>
<td>Meter</td>
</tr>
<tr>
<td>cm</td>
<td>Centimeter</td>
</tr>
<tr>
<td>mm</td>
<td>Millimeter</td>
</tr>
<tr>
<td>km</td>
<td>Kilometer</td>
</tr>
<tr>
<td>L</td>
<td>Liter</td>
</tr>
<tr>
<td>ml or mL</td>
<td>Milliliter</td>
</tr>
<tr>
<td>lb</td>
<td>Pound</td>
</tr>
<tr>
<td>oz</td>
<td>Ounce</td>
</tr>
<tr>
<td>kg</td>
<td>Kilogram</td>
</tr>
<tr>
<td>g</td>
<td>Gram</td>
</tr>
<tr>
<td>mg</td>
<td>Milligram</td>
</tr>
<tr>
<td>pt</td>
<td>Pint</td>
</tr>
<tr>
<td>qt</td>
<td>Quart</td>
</tr>
<tr>
<td>c</td>
<td>Cup</td>
</tr>
<tr>
<td>gal</td>
<td>Gallon</td>
</tr>
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### Formulas

<table>
<thead>
<tr>
<th>Purpose</th>
<th>Formula</th>
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<tbody>
<tr>
<td>Perimeter of rectangle</td>
<td>P = 2l + 2w</td>
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<tr>
<td>Area of a square</td>
<td>A = 4s</td>
</tr>
<tr>
<td>Area of a rectangle</td>
<td>A = l ∙ w</td>
</tr>
<tr>
<td>Volume of a cube</td>
<td>V = s³</td>
</tr>
<tr>
<td>Volume of a rectangular solid</td>
<td>V = l ∙ w ∙ h</td>
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<thead>
<tr>
<th>Símbolo</th>
<th>Significado</th>
<th>Ejemplo</th>
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<tr>
<td>△</td>
<td>Triángulo</td>
<td>△ABC</td>
</tr>
<tr>
<td>∠</td>
<td>Ángulo</td>
<td>∠ABC</td>
</tr>
<tr>
<td>→</td>
<td>Rayo</td>
<td>→AB</td>
</tr>
<tr>
<td>↔</td>
<td>Línea</td>
<td>↔AB</td>
</tr>
<tr>
<td>—</td>
<td>Segmento de recta</td>
<td>AB</td>
</tr>
<tr>
<td>⊥</td>
<td>Perpendicular a</td>
<td>AB ⊥ BC</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>&lt;</td>
<td>Menor que</td>
<td>2 &lt; 3</td>
</tr>
<tr>
<td>&gt;</td>
<td>Mayor que</td>
<td>3 &gt; 2</td>
</tr>
<tr>
<td>=</td>
<td>Igual a</td>
<td>2 = 2</td>
</tr>
<tr>
<td>°F</td>
<td>Grados Fahrenheit</td>
<td>100°F</td>
</tr>
<tr>
<td>ºC</td>
<td>Grados Celsius</td>
<td>32°C</td>
</tr>
<tr>
<td>⊥</td>
<td>Ángulo recto (90º ángulo)</td>
<td>⊥</td>
</tr>
<tr>
<td>...</td>
<td>Y así...</td>
<td>1, 2, 3, ...</td>
</tr>
<tr>
<td>×</td>
<td>Multiplica</td>
<td>9 × 3</td>
</tr>
<tr>
<td>·</td>
<td>Multiplica</td>
<td>3 · 3 = 9</td>
</tr>
<tr>
<td>÷</td>
<td>Divide</td>
<td>9 ÷ 3</td>
</tr>
<tr>
<td>+</td>
<td>Suma</td>
<td>9 + 3</td>
</tr>
<tr>
<td>−</td>
<td>Resta</td>
<td>9 − 3</td>
</tr>
<tr>
<td>➝</td>
<td>Dividido entre</td>
<td>3 ➝ 9</td>
</tr>
<tr>
<td>R</td>
<td>Residuo</td>
<td>3 R 2</td>
</tr>
<tr>
<td>%</td>
<td>Porcentaje</td>
<td>50%</td>
</tr>
<tr>
<td>x²</td>
<td>“x” cuadrada (por sí mismo)</td>
<td>3² = 3 × 3 = 9</td>
</tr>
<tr>
<td>x³</td>
<td>“x” cúbica</td>
<td>3³ = 3 × 3 × 3 = 27</td>
</tr>
<tr>
<td>√</td>
<td>Raíz cuadrada</td>
<td>√9 = 3 porque 3 × 3 = 9.</td>
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<table>
<thead>
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<th>Significado</th>
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<tr>
<td>pie</td>
<td>pie</td>
</tr>
<tr>
<td>pulg</td>
<td>pulgada</td>
</tr>
<tr>
<td>yd</td>
<td>yarda</td>
</tr>
<tr>
<td>mi</td>
<td>milla</td>
</tr>
<tr>
<td>m</td>
<td>metro</td>
</tr>
<tr>
<td>cm</td>
<td>centímetro</td>
</tr>
<tr>
<td>mm</td>
<td>milímetro</td>
</tr>
<tr>
<td>km</td>
<td>kilómetro</td>
</tr>
<tr>
<td>L</td>
<td>litro</td>
</tr>
<tr>
<td>ml or mL</td>
<td>mililitro</td>
</tr>
<tr>
<td>lb</td>
<td>libra</td>
</tr>
<tr>
<td>oz</td>
<td>onza</td>
</tr>
<tr>
<td>kg</td>
<td>kilogramo</td>
</tr>
<tr>
<td>g</td>
<td>gramo</td>
</tr>
<tr>
<td>mg</td>
<td>miligramo</td>
</tr>
<tr>
<td>pt</td>
<td>pinta</td>
</tr>
<tr>
<td>ct</td>
<td>cuarto</td>
</tr>
<tr>
<td>tz</td>
<td>taza</td>
</tr>
<tr>
<td>gal</td>
<td>galón</td>
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<th>Fórmula</th>
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<td>P = 2l + 2w</td>
</tr>
<tr>
<td>Área de un cuadrado</td>
<td>A = 4l</td>
</tr>
<tr>
<td>Área de un rectángulo</td>
<td>A = l · w</td>
</tr>
<tr>
<td>Volumen de un cubo</td>
<td>V = l³</td>
</tr>
<tr>
<td>Volumen de un sólido rectangular</td>
<td>V = l · w · h</td>
</tr>
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